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Possibility of estimating reliability of diesel engines by applying the theory of semi-Markov processes and making operational decisions by considering reliability of diagnosis on technical state of this sort of combustion engines

The paper presents semi-Markov models of technical state transitions for diesel engines, useful for determining reliability of the engines. A possibility of application of a three-state model with a simplified matrix function, or even a two-state model, to determine reliability of the engines, has been described herein on examples of known from literature semi-Markov models, for the case when appropriate diagnosing systems (SDG) are used to identify technical condition of the engines considered as diagnosed systems (SDN)). A risk function and a renewal (restitution) function have been proposed to apply for developing a two-state model of engine state transitions. An opportunity of considering reliability of diagnosis while making operational decisions by applying the statistical decision theory, has also been presented. Conditional probability $P(s_i/\mathbf{K}_i)$ *is recognized as a measure of reliability of diagnosis.*

Key words: *diagnostics, reliability, semi-Markov process, diesel engine, statistics, diagnosed system, diagnosing system, diagnostic system*

Możliwość oszacowania niezawodności silników o zapłonie samoczynnym z zastosowaniem teorii procesów semi-Markowa oraz podejmowania decyzji eksploatacyjnych z uwzględnieniem wiarygodności diagnozy o stanie technicznym tego rodzaju silników spalinowych

W artykule przestawiono semimarkowskie modele zmian stanów technicznych silników o zpałonie samoczynnym, przydatne do określenia niezawodności tych silników. Wykazano, na przykładzie znanych z literatury modeli semimarkowskich, że możliwe jest zastosowanie do określenia niezawodności tych silników modelu trójstanowego o uproszczonej macierzy funkcyjnej a nawet modelu dwustanowego, w przypadku zastosowania odpowiednich systemów diagnozujacych (*SDG*) *do identyfikacji stanu technicznego wspomnianych silników jako systemów diagnozowanych* (*SDN*). *Do opracowanie dwustanowego modelu zmian stanów silniów zaproponowano wykorzystanie funkcji ryzyka i funkcji intensywnosci odnowy (restytucji). Przedstwiono także możliwość uwzględnienia wiarygodności diagnozy przy podejmowaniu decyzji eksploatacyjnych w przypadku zastosowaia statystycznej teorii decyzji. Za miarę wiarygodności diagnozy przyjęte zostało prawdopodobieństwo warunkowe* $P(s_i/\mathbf{K}_i)$.

Słowa kluczowe: *diagnostyka, niezawodność, proces semimarkowski, silnik o zapłonie samoczynnym, statystyka, system diagnozowany, system diagnozujący, system diagnostyczny*

1. Introduction

In the phase of operating marine diesel engines, particularly the main ones (engines employed in propulsion systems of ships), it is important to plan and control their operation. Planning requires having a knowledge of engine operation reliability to be expected during performance of transport tasks by a ship. In order to control the engine operation, when implementing the approved plan of operation, it is significant to know a complete diagnosis (instantaneous diagnosis, prognosis and genesis) that enables prediction of not only the technical state, but also the loads which engines under operation can be subjected to.

Considering the definitions of reliability of machines, provided in many publications [18, 21, 26], reliability of diesel engines can be defined in a similar way, as capability of the engines to convert energy in full range of loads, which they were fit to in the designing and manufacturing phases. And the probability of proper energy conversion for all performances in defined time and determined operating conditions can be recognized as a measure of such understood reliability of the engines [2, 4, 12, 21, 26].

It can be assumed, like in publications [3, 6, 7], that any diesel engine works reliably if its technical condition can be classified to the class (set) of states of full ability and denoted as s_1. If the engine, due to its technical condition, cannot be loaded to the maximum extent, but only to the extent limited by the rated power value [12, 22], its state must be recognized as a state of partial ability (s_2). When the engine, due to its significant wear, can be loaded only to the extent limited by the continuous operating power value [12, 22], it should be assumed that its technical state does not satisfy operating requirements and must be regarded as a state of disability (s_3).

The mentioned technical states of this sort of engines and their time durations can be disclosed by using appropriate diagnosing systems (SDG), e.g. for marine engines, such as *CoCoS* (*Computer Controlled Surveillance System*) of *MAN* company, or *CBM* (*Condition-Based Maintenance*) of *Wartsila* company, videoskope of *Everest* company and others [20, 27, 28, 29]. Obtained through applying the technical diagnostics information on duration of state s_1 and the moment of losing it, as well as on the moments of occurring states s_2 and s_3, and their duration, enables application of the theory of semi-Markov processes for determining reliability of the engines [6, 7, 8]. When the systems *SDG* are so improved that they enable development of a complete diagnosis (*pDG*) comprising not only a reliable instantaneous diagnosis, but also prognosis of duration of state s_1, it is possible to obtain a transitions graph of engine states s_1, s_2, s_3, which is simplified in comparison to the graph described in publications [5, 7, 11, 25].

2. Possible semi-Markov models of engine state transitions

Building a semi-Markov model of a real process of technical state changes which proceed in the operating phase of a diesel engine, is a prerequisite for applying the theory of the semi-Markov processes. The properties of the models are as follows [4, 8, 19, 21, 23, 24]:

1) Markov condition is satisfied that future development of states of any engine (the process of technical state changes), for which the semi-Markov model was built, depends only on its state at the given time, not on engine functioning in the *past*, so that its future does not depend on the *past*, but on the *present*;
2) random variables T_i (denoting the time duration of state s_i regardless of which state is next) and T_{ij} (denoting the time duration of state s_i, provided that the next state of the process is state s_j) have distributions different than exponential.

In the case of marine main engines it can be recognized that the Markov condition is satisfied because the following hypothesis was proved in researches [3, 7]: *prognosing the technical state of any diesel engine at the time $\tau_n + t$, when only its state at the moment τ_n is known, is possible, because the engine state considered at any moment $\tau_n(n = 0, 1, ..., m;\ \tau_0 < \tau_1 < ... < \tau_m)$ indeed depends on the directly preceding state, not on the states that were before nor their time duration.*

Additionally, the studies show that random variables like time of proper operation (T_u) and renewal time (T_o) of this sort of engines can be described with gamma and normal distributions, and the Weibull-Gniedenko distribution as well [1, 2, 26].

Semi-Markov process is fully defined if its function matrix is known [3, 7, 18]

$$\mathbf{Q(t)} = [Q_{ij}(t)] \qquad (1)$$

whose non-zero elements are interpreted as follows:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1} - \tau_n < t \mid W(\tau_n) = s_i\} = \\ = p_{ij}F_{ij};\ s_i, s_j \in S;\ i, j = 1, 2, \ldots, n;\ i \neq j$$

where p_{ij} – probability of state transition from s_i to s_j, F_{ij} – distribution of random variable T_{ij}, and when the initial distribution is given

$$p_i = P\{W(0) = s_i\},\ s_i \in S;\ i = 1, 2, \ldots, n \qquad (2)$$

The paper [4] presents a model of state transition for diesel engines $\{W(t): t \geq 0\}$, whose values are elements of the set of classes (subsets) of technical conditions called states (which are of essential meaning in the operating practice):

$$S = \{s_i\,;\ i = 1, 2, 3, 4\} \qquad (3)$$

described as follows:

- s_1 - state of full (total) ability
- s_2 - state of partial (not full, not total) ability
- s_3 - state of disability for task,
- s_4 - state of full (total) disability.

Distinguishing states $s_i \in S$ ($i = 1, 2, 3, 4$) is very important for diesel engines as it is extremely significant to use them in state s_1, or s_2.

In this case the initial distribution of the process $\{W(t): t \geq 0\}$ is defined as follows:

$$p_1 = P\{W(0) = s_1\} = 1 \cap p_i = P\{W(0) = s_i\} = 0 \\ \text{for } i = 2, 3, 4 \qquad (4)$$

while its function matrix has the form:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 \\ Q_{31}(t) & 0 & 0 & Q_{34}(t) \\ Q_{41}(t) & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

The model considers situations where the user can take the risk to perform the task when the engine finds in state s_2 and even take the risk to perform some tasks when its state is s_3. In the operating practice, it is difficult to distinguish unambiguously state s_1 from state s_2. This refers particularly to the case of deteriorating technical condition of marine main engine. This is due to the fact that external conditions (height, speed and direction of wave, speed and direction of wind, layer of seaweed and shellfish on the underwater section of hull, speed and direction of ocean currents, etc.) under which the sea-going vessels are operated and tasks are performed by users of the engines, significantly affect engine load changes [12, 22, 29]. In this operating situation, applying the diagnosing systems (*SDG*) to identify technical conditions of shipborne main engines as diagnosed systems (*SDN*) is easier when developing a tree-state set of technical states [4, 12, 22, 29]:

$$S = \{s_1, s_2, s_3\} \tag{6}$$

with interpretation:

- s_1 – state of engine full ability, which is recognized when the engine can be loaded in full range which was fit to in the designing and manufacturing phases,
- s_2 – state of engine partial ability, which occurs in the moment and proceeds when the engine cannot be loaded in the full range of engine performances, but can be loaded within the field not smaller than the range limited by the continuous power characteristic, however with no possibility of loading in the field of rotational speed overloads.
- s_3 – state of engine disability, which occurs and proceed when the engine can be loaded only in the range below the continuous power characteristic, but also with no possibility of loading in the field of rotational speed overloads.

The described technical states $s_i \in S(i = 1, 2, 3)$ are values of the process $\{X(t): t \geq 0\}$of state transitions, with initial distribution, defined by the formula:

$$P_i = P\{X(0) = s_i\} = \begin{cases} 1 & dla \quad i = 1 \\ 0 & dla \quad i = 2,3 \end{cases} \tag{7}$$

and following function matrix:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \tag{8}$$

Application of an *SDG* enabling development of a reliable diagnosis [13, 15], allows implementation of the operation strategy, which includes preventive maintenance of the main engine when its state is recognized as s_2. This prevents damage to the engine during its work. Therefore, the stochastic process $\{X(t): t \geq 0\}$, as a model of the process of engine state transitions can be simplified due to $p_{23} = 0$, and thus there is no function $Q_{23}(t)$ (8).

3. Simplified semi-Markov models of engine technical state transition

Currently applied diagnosing systems (*SDG*) for identifying the technical condition of marine main engines considered as diagnosed systems (*SDN*), such as Cocos (Con-trolled Computer Surveillance System) of MAN company, or CBM (Condition-Based Maintenance) of *Wartsila* company [27, 28, 29] are designed to disclose the most important states classified as s_2 for the engines [7, 9, 10, 11, 23]. It can be assumed that occurrence of state s_2, is a result of damage to the engine. Such sort of damage allows further operation of the engine, but does not ensure performance of the task Z_d which must be carried out. Disclosure by the *SDG* of the state s_2 in the engine enables performance of some adequate preventive service and full engine recovery, and in consequence regaining of state s_1, that allows performance of the task Z_d. Despite using the *SDG*, damages to engines are reported during their operation. The technical state that results from such damage is recognized as s_3

When $p_{23} = 0$, the model of the process of state transitions for main engines is a stochastic process $\{Y(t): t \geq 0\}$ with the set of states $S = \{s_i ; i = 1, 2, 3\}$ (6) and the graph of state transitions as depicted in Fig. 1. Transitions of the states proceed at the moments $\tau_0 = 0$, τ_1, τ_2, τ_3, τ_4, τ_5, …, τ_n (Fig. 2). Due to the fact that the process $\{Y(t): t \geq 0\}$ is a semi-Markov process, the moments are random variables which satisfy the condition:

$$\begin{aligned} &P\{Y(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < \tau | Y(\tau_n) = s_i, Y(\tau_{n-1}), \ldots \\ &\ldots Y(\tau_1), Y(\tau_0), \tau_n - \tau_{n-1}, \ldots, \tau_1 - \tau_0, \tau_0\} = \\ &= P\{Y(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < \tau | Y(\tau_n) = s_i\} \end{aligned}$$

A graph of state transitions of the process $\{Y(t): t \geq 0\}$ and thus also of engine, is shown in Fig. 1. This model of changing technical states of engine is a simplified model when comparing to the model in the form of the process $\{X(t): t \geq 0\}$. The simplification consists in that the function matrix (8) of the process $\{Y(t): t \geq 0\}$ does not take into account the function $Q_{23}(t)$, because $Q_{23}(t) = 0$, due to the fact that $p_{23} = 0$.

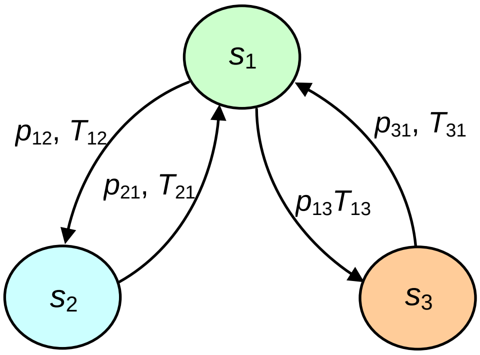

Fig. 1. Graph of engine state transitions: p_{ij} – probability of engine transition from state s_i to state s_j, T_{ij} – duration of state s_i if the process transits to state s_j; $i \neq j$; $i, j = 1, 2, 3$

An example of the process $\{Y(t)\colon t \geq 0\}$ is illustrated in Fig. 2

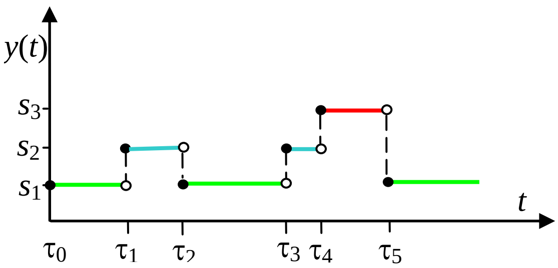

Fig. 2. An example of the process $\{Y(t)\colon t \geq 0\}$for an engine: $\{y(t)\colon t \in T\}$ – process of technical state transitions, t – operating time; s_1 – state of full ability, s_2 – state of partial ability, s_3 – state of disability

Initial distribution of the process is defined by the formula (7), while its function matrix is as follows:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & 0 \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \qquad (9)$$

The limiting distribution for the process $\{X(t)\colon t \geq 0\}$ can be derived from the formula [2, 11, 19, 24]:

$$P_j = \frac{\pi_j \cdot E(T_j)}{\sum_{k=1}^{3} \pi_k \cdot E(T_k)}, \quad j = 1,\ 2,\ 3 \qquad (10)$$

The distribution $\pi_j (j = 1, 2, 3)$ in the formula (10) is a limiting distribution of the Markov chain $\{Y(\tau_n)\colon n = 0, 1, 2, 3, \ldots\}$embedded in the process $\{Y(t)\colon t \geq 0\}$. This distribution, as it follows from the function matrix (9), satisfies the system of equations [19]:

$$\left.\begin{aligned} &[\pi_1, \pi_2, \pi_3] \cdot \begin{Bmatrix} 0 & p_{12} & p_{13} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{Bmatrix} = [\pi_1, \pi_2, \pi_3] \\ &\pi_1 + \pi_2 + \pi_3 = 1 \end{aligned}\right\} \qquad (11)$$

The solution of the system of equations (11), when considering the formula (10), are the probabilities:

$$P_1 = \frac{E(T_1)}{H}, \quad P_2 = \frac{p_{12} \cdot E(T_2)}{H}, \quad P_3 = \frac{p_{13} \cdot E(T_3)}{H} \qquad (12)$$

while

$$H = E(T_1) + p_{12} \cdot E(T_2) + p_{13} \cdot E(T_3)$$

where:

P_1, P_2, P_3 – probabilities that diesel engine finds respectively in the states: s_1, s_2, s_3;

π_j – limiting probability of a Markov chain embedded in the process$\{Y(t)\colon t \geq 0\}$ that describes possibility of occurring state s_j, $j = 1, 2, 3$;

p_{ij} – probability of the process $\{Y(t)\colon t \geq 0\}$ transition from state s_i to state s_j;

$E(T_j)$ – expected value of duration of state s_j.

When performance of the task by the main engine is possible only if it finds in the state of full ability (i.e. state s_1), its reliability is defined by the probability P_1. However, when the task can be performed by the main engine, even when it finds in a state of partial ability (s_2), the reliability of the engine can be determined by the sum of probabilities of the two types of states.

Depending on the selected operation strategy, the even more simplified model of state transitions can be applied for main marine engines. As the ship safety is essential when performing the transport task, there are considered only two states: s_1 (state of full ability) and s_{2*} (state of disability), where $s_{2*} \equiv s_2 \cup s_3$. In such case, the process $\{Z(t)\colon t \geq 0\}$ with a two-element set of states: s_1 and s_{2*} will be the model of engine state transitions. For such a simple model of engine state transitions, the intensity function $\lambda_{ij}(t)$ of transition of the process $\{Z(t)\colon t \geq 0\}$ (so also transition of the engine) from s_i to $s_j (i \neq j;\ i, j = 1, 2^*)$ can be applied for calculating probabilities P_1 and P_2. Transition intensity functions $\lambda_{ij}(t)$ are the ratios of transition probabilities p_{ij}, referred to the time interval Δt, which they concern. Thus, the graph of state transitions of the process $\{\{Z(t)\colon t \geq 0\}$, thus also the engine, has the form as depicted in Fig. 3.

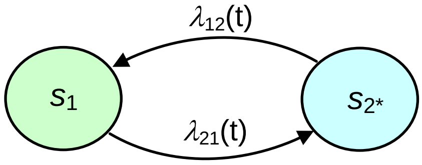

Fig. 3. Graph of engine state transitions:
λ_{ij} – probability of the process transition from stare s_i to stare s_j; ($i \neq j$; $i, j = 1, 2^*$)

Due to the fact that the function $\lambda_{ij}(t)$ is a function of time t, thus dependent on Δt, therefore the relationship below is valid:

$$\lambda_{ij} = \lim_{\Delta t \to 0} \frac{\lambda(t)}{\Delta t} = const \tag{13}$$

The assumption in considerations that $\lambda_{ij} = const$ means that the process $\{Z(t): t \geq 0\}$ is a discrete-state continuous-time Markov model. This further means that random variables, which are the time of proper work of engine and the time of its renewal, are recognized as random variables with exponential distributions. Consideration of the time of proper work of engine as a random variable with exponential distribution is justified by the fact that in stormy weather conditions engines are subject to impact loads (so-called shock pulses) in particular [2, 26]. Also probabilities of performing renewal to a damaged engine in time of storm are small and decrease with extending the time needed to do the renewal, which also makes it possible to assume that the renewal time is a random variable with exponential distribution [26]. Furthermore, the exponential distribution significantly reduces the values of engine reliability over time when comparing to other distributions of this sort of continuous random variables. In the proposed model, the intensity λ_{12*} of the process transition from state s_1 to state s_{2*} is called the risk function, and the transition intensity λ_{2*1} – intensity function of renewal (restitution). In this case the probabilities P_1 and P_2 can be derived from the formulas [17]:

$$P_1(t) = \frac{\lambda_{2*1}}{\lambda_{2*1} + \lambda_{12}} + \frac{\lambda_{12}}{\lambda_{12} + \lambda_{2*1}} \exp\{-(\lambda_{12} + \lambda_{2*1})t\}$$

$$P_{21}(t) = \frac{\lambda_{12}}{\lambda_{12} + \lambda_{12*}} - \frac{\lambda_{12}}{\lambda_{12} + \lambda_{2*1}} \exp\{-(\lambda_{12} + \lambda_{2*1})t\} \tag{14}$$

When the operating time t of engine is very long (in theory $t \to \infty$), the formulas (14) take the forms:

$$P_1(t) = \frac{\lambda_{2*1}}{\lambda_{2*1} + \lambda_{12}} \text{ oraz } P_{21}(t) = \frac{\lambda_{12}}{\lambda_{12} + \lambda_{12*}} \tag{15}$$

since in the formulas (16) the values exp{•}= 0.

When transportation tasks are carried out by ships in favorable conditions, so in the periods (seasons) in which there are no storms, it can be assumed that the random variable which is the time of proper operation of the engine, has a gamma distribution, and the random variable which is the engine renewal time - normal distribution. Then, it can be assumed that [2]:

$$\lambda_{12}(t) = \frac{\lambda_{12}^r t^{r-1}}{(r-1)!\left[1 + \frac{1}{1!}\lambda_{12}t + \frac{1}{2!}(\lambda_{12}t)^2 + \ldots + \frac{(\lambda_{12}t)^{r-1}}{(r-1)!}\right]} \tag{16}$$

where: r – parameter of distribution shape.

With the growth of t ($t \to \infty$) the function (16) increases monotonically to $\lambda_{12} = const$ [2, 26]. The function λ_{2*1}, whereas, is defined by the formula [2]:

$$\lambda_{2*1}(t) = \frac{1}{\sqrt{2\pi}\sigma\Phi\left(\frac{m_1 - t}{\sigma}\right)} \exp\left\{-\frac{(t - m_1)^2}{2\sigma^2}\right\} \tag{17}$$

where:

σ – standard deviation, m_1 – zero moment of the first order (expected value),

$\Phi\left(\frac{m_1 - t}{\sigma}\right)$ – Laplace function.

With the growth of t the function (17) increases monotonically [2, 26].

The probabilities defined by the formulas (12), (14) and (15) are significant in the phase of planning the engine operation whose implementation requires securing the funds, fuel supply and lubricating oil, and spare parts. However, the phase of engine operating requires controlling the process of changes in engine technical condition, which consists in making decisions. This requires knowledge of reliability of diagnosis on the engine technical state and consequences of making a decision selected from among possible decisions in the given operating situation for this kind of engines [14, 15, 16]. Optimal decisions can be made when applying semi-Markov decision (controlled) processes or statistical decision theory. From among the theories, Bayesian statistical decision theory is easier for use. In this theory a criterion of decision-making is an expected value of decision consequences. According to this criterion, making an operating decision consists in selecting the optimal decision belonging to the set of possible decisions in the given operating situation. Such a decision is always this one which the highest expected value of consequences corresponds to [1]. Application of Bayesian statistical decision theory requires development of a statistical model of decision-making.

4. Statistical model of decision-making

Application of an appropriate diagnosing system (*SDG*) for identifying technical condition of a main engine as diagnosed system (*SDN*), allows obtaining a diagnosis on technical state of the engine, with defined reliability. A measure of diagnosis reliability is the conditional probability $P\{s_i/\mathbf{K_i}\}$, that the engine is in state s_i provided that the value vector $\mathbf{K_i}$ of diagnostic parameters corresponding to this state, is observed [13, 15]. This information and also the knowledge of consequences of each decision, enable application of the statistical decision theory for making a rational decision, e.g. from the following two possible [14, 16]:

- decision d_1 – perform first an adequate preventive maintenance of engine to renew its state, which is indispensable to carry out the task Z_d, and then, start performing the task according to the schedule set by the customer.
- decision d_2 – do not perform the maintenance service and start performing the ordered task Z_d.

The Bayesian statistical decision theory shows that in such a decision situation, e.g. for the process $\{Z(t)$: $t \geq 0\}$ of changing engine technical states, considerations should include decision dendrite as demonstrated in Fig. 4.

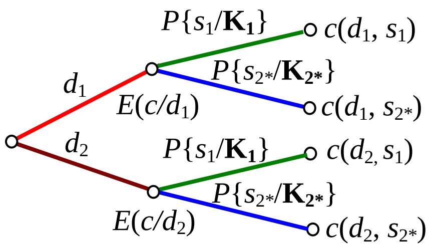

Fig. 4. Decision dendrite for making decision d_1 or d_2: d_1 – decision to perform first the appropriate preventive maintenance to the engine and then the given task, d_2 – decision to perform the ordered task without prior preventive maintenance to the engine,
$P\{s_1/\mathbf{K_1}\}$ – probability that engine is in state s_1, provided that vector $\mathbf{K_1}$ of diagnostic parameters is observed,
$P\{s_{2*}/\mathbf{K_2}\}$ – probability that engine is in state s_{2*}, provided that vector $\mathbf{K_2}$ of diagnostic parameters is observed,
$c(d_1, s_1)$ – consequence of decision d_1 when engine state is s_1, $c(d_1, s_{2*})$ – consequence of decision d_1 when engine state is s_{2*}, $c(d_2, s_1)$ – consequence of decision d_2 when engine state is $s_{1,}$, $c(d_2, s_{2*})$ – consequence of decision d_2 when engine state is s_{2*}

The decision dendrite provided in Fig. 4 indicates that the expected values can be derived from the following relationships [14, 15]:

$$\left.\begin{aligned} E(c|d_1) &= P(s_1|\mathbf{K_1})c(d_1, s_1) + P(s_{2*}|\mathbf{K_2})c(d_1, s_{2*}) \\ E(c|d_2) &= P(s_1|\mathbf{K_1})c(d_2, s_1) + P(s_{2*}|\mathbf{K_2})c(d_2, s_{2*}) \end{aligned}\right\} \tag{18}$$

where the below relationship is valid:

$$P(s_1|\mathbf{K_1}) + P(s_{2*}|\mathbf{K_{2*}}) = 1$$

In accordance with the decision-making rule, decision d_1 should be made when $E(c|d_1) > E(c|d_2)$, and inversely – decision d_2. should be made when $E(c|d_1) < E(c|d_2)$.

Probabilities $P(s_i/\mathbf{K_i})$ as measures of reliability of diagnosis can be derived from the dependence [14]:

$$P(s_i|\mathbf{K_i}) = \frac{P(A)P(s_i|A)P(\mathbf{K_i}|A \cap s_i)}{P(\mathbf{K_i})P(A|\mathbf{K_i} \cap s_i)} \tag{19}$$

where: $P(s_i/\mathbf{K_i})$ – probability that engine is in state $s_i(i = 1, 2^*)$, provided that the value vector $\mathbf{K_i}$ of diagnostic parameters is observed, A – an event that *SDG* works properly, P(A) – probability of proper work of *SDG*, $P(s_i/A)$ – probability that engine is in state $s_i(i = 1, 2^*)$, provided that *SDG* works properly, $P(\mathbf{K_i}/A \cap s_i)$ – probability that the value vector $\mathbf{K_i}$ of diagnostic parameters is observed, provided that *SDG* works properly and the engine is in state s_i, $P(\mathbf{K_i})$ – probability that the value vector $\mathbf{K_i}$ of diagnostic parameters is observed, $P(A/\mathbf{K_i} \cap s_i)$ – probability that *SDG* works properly, provided that the value vector $\mathbf{K_i}$ of diagnostic parameters is observed and the engine is in state s_i

Assuming that $P(A) = 1$, the formula (19) can be simplified to the following form [14]:

$$P(s_i / \mathbf{K_i}) = \frac{P(s_i)P(\mathbf{K_i} / s_i)}{P(\mathbf{K_i})} \tag{20}$$

which allows obtaining a measure of diagnosis accuracy.

Considering the measures of diagnosis reliability (19) or accuracy (20) in engine diagnostics allows development of a rational operating diagnosis on engine ability to perform the task Z_d [5, 16].

5. Final remarks and conclusions

In research on reliability of engines a variety of mathematical models can be used, including more adequate functional models in the form of semi-Markov processes. Semi-Markov processes are increasingly applied, and not only for solving different issues regarding reliability and diagnostics of diesel or other combustion engines.

Application of a semi-Markov process as a model of changes in the mentioned reliability states of a main engine at defined time, results from that the random variable $T_{(ij)}$ denoting duration of state

$s_{(i)}$, provided that the successive state is s_j, and the variable random T_i denoting duration of engine state $s_{(i)}$ (e.g. i = 1, 2, 3 or i = 1, 2^*), regardless of which state is successive, have distributions that belong in the set $R_+ = [0, +\infty)$.

When using an appropriate *SDG* enabling development of a complete diagnosis (instantaneous diagnosis, prognosis and genesis) with defined reliability $P(s_i/\mathbf{K_i})$, it makes sense to apply a two-state Markov process for studying this kind of engines, because in this case that includes additional requirement of ensuring a high safety level to a sea-going ship, it can be assumed that the random variables $T_{(ij)}$ and $T_{(i)}$ have exponential distributions.

The presented models can be of significant practical meaning due to the ease of defining the estimators of transition probabilities $p_{(ij)}$ and the ease of estimating the expected values $E(T_{(j)})$. Thereby, it should be considered that the point estimate of the expected value $E(T_{(j)})$ does not allow determining the accuracy of the estimation. Such accuracy is possible to provide by interval estimation, where the confidence interval $[t_{d(j)}, t_{g(j)}]$, with random limits is determined, which contains with a defined probability (confidence level) β, the unknown expected value $E(T_{(j)})$.

Moreover, consideration of reliability of the diagnosis $P(s_i/\mathbf{K_i})$ in the operating phase enables, by using the statistical decision theory, to make optimal operating decisions from among the possible decisions for main engines in the given operating situation.

It can reasonably be expected that the proposed models may also be useful in studies on reliability of other machines.

Abbreviations and terms

pDG – complete diagnosis on engine technical state,

SD – diagnostic system, *SDN* - diagnosed system

SDG – diagnosing system, $P(s_i/\mathbf{K_i})$ – probability being a measure of diagnosis reliability

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