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Vehicle exhaust emissions in the light of modern research tools: synergy of chassis dynamometers and computational models

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Assessing vehicle emissions is crucial for understanding their environmental impact and developing effective emission reduction strategies. This article discusses modern research tools that combine traditional laboratory measurements on chassis dynamometers with advanced theoretical models. Probabilistic methods, including stochastic processes based on Markov and semi-Markov chains, are important tools for modelling driving cycles, considering the variability of road conditions and driver behaviour. The article also presents mathematical approaches to emission data analysis, considering both the states of the technical system and the transitions between them, which allows for precise modelling of real vehicle operating conditions. Ultimately, the synergy of experimental measurements with computational modelling offers a more complete and accurate tool for assessing pollutant emissions, which is crucial in global efforts to improve air quality.

Key words: *driving test, pollutant emission, combustion engine, pollutant emission characteristics, Markov and semi-Markov process*

1. Introduction

The dynamic development of motorisation is associated with the increasing impact of road transport on the environment. Emissions of pollutants from motor vehicles pose a significant challenge for scientists and decision-makers responsible for environmental policy. Many decision-making bodies would like to obtain precise data on the actual emissions of pollutants and fuel consumption of combustion engines, but achieving this goal is difficult [4].

Vehicle emissions are determined by many factors, including driving style, road conditions, fuel type and quality, and emission control technologies [3, 17]. Vehicle emissions depend on the nature of the vehicle and the interaction of factors such as traffic conditions [24], driving style [10], fuel quality and specifications [17, 21], the technology behind the vehicle design, such as ambient conditions [19], and emission control technology [3]. These factors determine the number and quantity of pollutants emitted during driving and cannot be replicated using engine test cycles.

Numerous driving tests have been developed in response to these limitations that simulate real-world vehicle operating conditions. Emissions tests are conducted on chassis dynamometers and real-world driving conditions under controlled laboratory conditions. Chassis dynamometers precisely measure engine performance and emissions parameters in various scenarios, reflecting selected driving profiles. Their advantage is that they can control speed, load, and weather conditions. However, these tests are limited because they do not fully represent the changing operating conditions and driver behaviour [3].

In real traffic conditions, tests are conducted using mobile emission analysers (PEMS – portable emissions measurement system) [3, 21, 22], which record pollutant emissions during regular vehicle operation. This method allows for more realistic results, considering various environmental and operational factors, but its accuracy may be limited by the lack of full control over all parameters. These tests, reflecting recorded speed curves and their synthetic equivalents, form the basis of homologation tests. Despite their standardisation, they are insufficient to assess transport's impact on the environment [25]. This is due to the complexity and unpredictability of combustion engine operating states, resulting from, among other things, differences in drivers' driving styles. Therefore, more and more attention is being paid to analytical methods that consider the stochastic nature of operational processes. Among such methods, the Monte Carlo method [4, 8, 9, 13], widely used to model road pollutant emissions, occupies an important place. By generating data based on random realisations of stochastic processes, this method allows for estimating the impact of various operating conditions on emissions. However, its disadvantages, such as high input data requirements, time consumption and error sensitivity, limit its practical application. Moreover, the Monte Carlo method [8] does not consider sequential time dependencies in the data, which can lead to inaccuracies in representing dynamic changes in real vehicle operating conditions.

The dependability of evaluating engine performance relies on the meticulous choice of an ideal prediction model for exhaust emission characteristics and the recognition of those parameters most conducive to precise prediction in single-cylinder four-stroke engines. Žvirblis et al. [26] revealed that although the chosen ideal model attained great accuracy in predicting most emission characteristics, nitrogen oxides (NO_x) emissions proved particularly resistant to precise forecasting.

The VALLUM01 tool, incorporating an artificial neural network (ANN), was designed to simulate and predict exhaust emissions and motor performance. It was tested for its applicability in evaluating diesel engine efficiency. The tool's intuitive interface facilitated efficient training, validation, and prediction processes [12]. The model's capacity to adapt to various fuel types while preserving predictive accuracy is pivotal for advancing environmental monitoring and ensuring compliance with stringent emission regulations. Through visual representations like histograms and scatter plots, the study emphasises the reliability and precision of the model, offering valuable insights into optimising diesel engine performance, refining fuel selection, and shaping operational strategies [15].

In light of these limitations, semi-Markov processes [6, 7] offer a modern alternative, allowing for more precise modelling of road pollutant emissions. They allow for considering transitions between different vehicle operating states, which is crucial for accurately representing changing operating conditions. This makes it possible to generate detailed distributions of pollutant emissions in real operating conditions, which is an important step towards a better understanding of the impact of road transport on the environment.

The dependencies of emissions on average speed estimated in this way can be used, for example, to create models for estimating pollutant emissions. Currently, many models for estimating pollutant emissions support laboratory and field studies. The most well-known models include COPERT (Computer Programme to Calculate Emissions from Road Transport) [15] and HBEFA [5] (Handbook Emission Factors for Road Transport). COPERT, developed by the European Environment Agency, is based on input data such as vehicle type, mileage, fuel type and driving profile. This model is widely used for emission inventories in Europe. HBEFA, on the other hand, offers detailed emission factors for different types of vehicles and driving scenarios, making it a valuable tool for regional studies.

In addition, integrating results from chassis dynamometer tests, real traffic conditions, and modelling creates the possibility of a more comprehensive approach to analysing pollutant emissions. Models such as COPERT and HBEFA can be calibrated using real data, which increases their precision and usefulness in the analysis of the impact of road transport on the environment.

By developing methods such as semi-Markov processes, scientists can contribute to creating more effective emission analysis tools and support informed decision-making in environmental policy and the design of green transport solutions. Additionally, in the context of applying modern analytical tools such as semi-Markov processes or road emission models (COPERT, HBEFA), responsible research and innovation play an increasingly significant role in supporting enterprises in the development of sustainable operational processes [14]. This article presents the application of semi-Markov processes to determine road emission distributions in real vehicle use conditions, analysing their potential and disadvantages compared to other methods. This article presents the application of semi-Markov processes to determine road emission distributions in real vehicle use conditions. It also includes tests on chassis dynamometers, analyses in real traffic conditions and the use of existing emission estimation models, which allows for a comprehensive assessment of pollutant emissions from road transport.

2. Research problem

Road emissions, an important indicator of the ecological properties of vehicles, determine the mass of pollutants emitted per unit of distance travelled. This is a dynamic parameter, strictly dependent on the engine operating conditions, which can be characterised by the following factors: engine load, engine speed, engine thermal state, including temperatures of individual components and engine systems (e.g. coolant and lubricating oil) and ambient conditions [1, 23].

Each of the above parameters significantly impacts the amount of pollutants emitted, which emphasises the complex nature of assessing the impact of road transport on the environment. The dynamics of road emissions can be described using operator relations that consider the complexity of the processes occurring during engine operation [3, 4, 8].

The operating state of an internal combustion engine in traction operation is determined by factors such as vehicle speed, motion resistance and the specifics of the vehicle design, including the configuration of the drive system [3, 20].

For engine operating conditions with a stable thermal state and comparable driving conditions, pollutant emission is largely determined by the vehicle speed profile. Therefore, traffic models that represent speed profiles play a key role in estimating emissions [3, 20]. A similar analytical approach can be observed in studies addressing the transport of materials and its impact on energetic efficiency, highlighting the importance of complex production and logistics processes in evaluating emissions [16].

To better understand the relationship between emissions and speed profiles, it is necessary to determine the point characteristics of these profiles. This approach allows for the analysis of the dependence of road emissions on representative parameters, such as the arithmetic mean value of speed. Alternatively, other indicators can be used, such as the mean value of the acceleration module or the speed and acceleration product module. These characteristics provide the basis for a more comprehensive analysis of emissions in various vehicle traffic scenarios. To determine the pollutant emission characteristics, the speed profiles of vehicles in homologation driving tests.

A passenger car with a spark-ignition engine with a displacement of 1798 cm^3 was used for the tests (Fig. 1).

Fig. 1. A research stand with a test object

Analytical tests were carried out: the first was conducted in real city traffic conditions, with the measurement route covering 12 kilometres from Plac Wilsona to Galeria Mokotów in Warsaw, and then repeated on a vehicle dynamometer. The map presenting the measurement route was developed using Google Maps (Fig. 2).

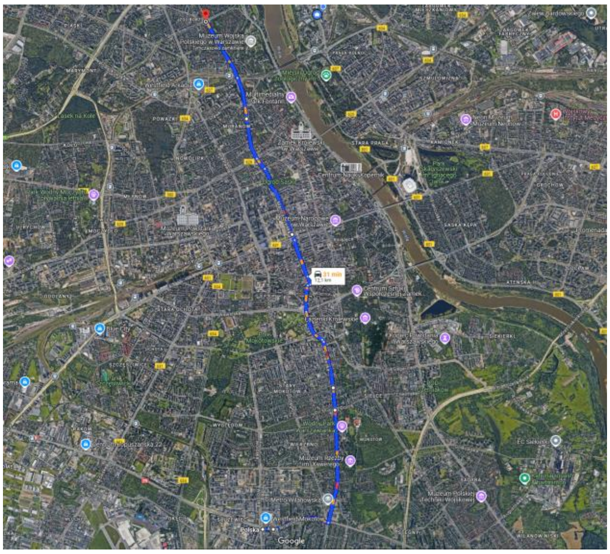

Fig. 2. Measurement route

The measurements were taken with the Semtech DS test equipment (Fig. 3). The Sensors Inc. analyser, type Semtech DS, measures emissions in real road traffic conditions, modelled on a chassis dynamometer. The set of flow meters in the device allows for measuring exhaust emissions from most engines that power motor vehicles.

Fig. 3. Test vehicle with Semtech DS test equipment installed

The test was based on a synthetic test eproducing a cycle of urban traffic. Pollutant emission measurements were performed using Semtech DS equipment in real road conditions. Then, the same cycle was reproduced on a chassis dynamometer, where pollutant emission measurements were again performed. For the measurement of methane, an analyser manufactured by AVL Emission Test Systems GmbH, type AVL FID i60 LHD GV0596, was used.

The results obtained from the empirical tests included parameters such as vehicle speed, acceleration, pollutant emission and fuel consumption. This article used only the results for methane (CH_4) and nitrogen oxide (NO) emissions.

3. Methodology

3.1. Semi-Markov process

Let $(\Omega, \mathcal{F}, P)$ be a probability space, the finite set of states will be denoted as $S = \{s_1, s_2, \cdots, s_k\}$, $\mathbb{N}$ and $\mathbb{R}$ are the sets of natural numbers, including zero and real numbers, respectively. We usually model the behaviour of a technical system by a stochastic process, i.e. sequence of random variables $\{X_t\}_{t\in\mathbb{R}}$, and then the realisations of these random variables belong to the set S, $X_t: \Omega \to S$. $\{X_t\}_{t\in\mathbb{R}}$ process is called a continuous-time stochastic process, but $\{X_t\}_{t\in\mathbb{N}}$ – discrete-time stochastic process.

Markov chains [6] are often used to describe the behaviour of technical systems.

Definition 1. (Markov process) The discrete-time stochastic process $\{X_t\}_{t\in\mathbb{N}}$ is called a Markov chain if the property

$$P(X_n = x_{i_n} | X_{n-1} = x_{i_{n-1}}, X_{n-2} = x_{i_{n-2}}, \dots, X_0 = x_{i_0}) = P(X_n = x_{i_n} | X_{n-1} = x_{i_{n-1}}). \quad (1)$$

is satisfied for any $n \in \mathbb{N}$ and states $x_{i_0}, x_{i_1}, \dots, x_{i_n} \in S$.

In many cases, the transition probability $P(X_{n+1} = x_j | X_n = x_i) = p_{ij}(n)$ from state $x_i \in S$ to state $x_j \in S$ does not depend on moment n, i.e. $p_{ij}(n) = p_{ij}$ for any moment $n \in \mathbb{N}$. Such a stochastic process is called a homogeneous Markov chain.

To describe the behaviour of a technical system based on a homogeneous Markov chain [6, 7] we estimate the transition probability matrix $P = [p_{ij}]_{i,j=1,2,\dots,k}$ between states. For $\{x_t\}_{0\le t\le n}$ sequence of realisation of Markov chain, where $x_t \in S$ for $0 \le t \le n$. For $i = 1,2,\dots,k$ we determine the number of times the system was in the state s_i as $n_i = \#\{t: x_t = s_i, 0 \le t \le n\}$ and $\sum_{i=1}^{k} n_i = n$. Additionally, value $n_{ij} = \#\{t: x_t = s_i, x_{t+1} = s_j, 0 \le t \le n-1\}$ for $1 \le i,j \le k$ denotes the number of transitions from state s_i to state s_j and $\sum_{j=1}^{k} n_{ij} = n_i$. The value of transition probability form state s_i to state s_j we designate as $\hat{p}_{ij} = \frac{n_{ij}}{n_i}$ for $1 \le i,j \le k$, thus matrix $P = [\hat{p}_{ij}]_{1\le i,j\le k}$ is the estimated transition probability matrix of homogeneous Markov chain. The transition probability from state x_i to state x_j in $m > 0$ steps for homogeneous Markov chain is equal $P(X_{n+m} = x_j | X_n = x_i) = p_{ij}^m$. Let $p(0) = (p_1(0), p_2(0), \dots, p_k(0))$, where $p_i(0) = P(X_0 = x_i)$ for $x_i \in S$ and $0 \le p_i(0) \le 1$, $i = 1,2,\dots,k$ with condition $\sum_{i=1}^{k} p_i(0) = 1$ is the initial distribution of X_0 random variable. When the initial distribution $p(0)$ of X_0 a random variable is known, then the probability distribution $p(n) = (p_1(n), p_2(n), \dots, p_k(n))$ of X_n random variable describing the state of the system at the moment n (distribution states after n transitions) is determined by the formula:

$$p(n) = p(0)P^n \quad (2)$$

where $[p_{ij}^m]_{1\le i,j\le k} = P^m$.

A semi-Markov process was used to identify the behaviour of vehicle travel in the city. In the presented research semi-Markov is a continuous-time stochastic process, where the sojourn time distribution in a state depends on the current and the future states. From above, we describe the behaviour of vehicle travel as a sequence of states by applying a Markov chain and analyse the sojourn time in states (time spent in state) [6, 7].

Definition 2. (Semi-Markov process) [7] The right-continuous and piecewise constant stochastic process $\{X_t\}_{t\geq 0}$ is called a semi-Markov process, if:

1. the sequence of random variables $\{X_{t_n}\}_{n\in\mathbb{N}}$ for $t_0 < t_1 < t_2 \ldots$ ($t_n \in \mathbb{R}$, $n \in \mathbb{N}$) is a homogeneous Markov chain with transition probability matrix $P = [p_{ij}]_{1\leq i,j\leq k}$, where $p_{ij} = P(X_{t_{n+1}} = x_j | X_{t_n} = x_i)$ and $i, j \in \mathbb{N}$;
2. the distribution of sojourn time $\tau_n = t_{n+1} - t_n$, $n = 1,2,\ldots$, a time when the process spent in the current state x_i from moment t_n to moment $t_{n+1} = t_n + \tau_n$ and jump to the future state x_j at the moment t_{n+1} depends on the current state and the future states, i.e. the probability $P(\tau_n \leq t | X_{t_n} = x_i, X_{t_{n+1}} = x_j)$ depends on only states x_i and x_j.

Below, it was assumed that $X_{t_n} = X_n$ for $n \in \mathbb{N}$. Thus semi-Markov process $\{(X_n, t_n)\}_{n\in\mathbb{N}}$ is a pair process characterised by both embedded Markov chain $\{X_n\}_{n\in\mathbb{N}}$ and the sojourn time stochastic process $\{\tau_n\}_{n\in\mathbb{N}}$ corresponding Markov chain, where $t_n = t_0 + \sum_{j=1}^{n} \tau_j$ for $n \in \mathbb{N}$.

3.2. Sojourn time distribution

A Weibull distribution [7] was used to analyse sojourn time, where the density function is given as follows

$$f(x, a, b) = \frac{a}{b}(x/b)^{a-1} e^{-(x/b)^a}, \quad x > 0 \tag{3}$$

but the distribution function

$$F(x, a, b) = 1 - e^{-(x/b)^a}, \quad x > 0 \tag{4}$$

where shape parameter $a > 0$ and scale parameter $b > 0$.

For semi-Markov processes at the moment t_n system transitions to state x_i and spends in this state τ_n time, but at the moment $t_{n+1} = t_n + \tau_n$ it transitions to the next state x_j, $1 \leq I, j \leq k$. Based on the sequence of realisation of sojourn times $\{\tau_n\}_{0\leq n\leq m}$ and corresponding them $\{x_n\}_{0\leq n\leq m}$, $x_n \in S$ sequence of states, which is Markov chain, first we determine the subsequences $T_{ij} = \{\tau_l : x_{t_l} = s_i, \quad x_{t_{l+1}} = s_j, \quad 0 \leq l \leq m\}$. The T_{ij} subsequence denotes the sequence of realisations of sojourn times when the system had s_i state and moved to state s_j, $1 \leq i, j \leq k$.

If $T_{ij} \neq \emptyset$, then for a sojourn time of the system in state s_i and transition to state s_j we estimate a_{ij} and b_{ij} parameters of Weibull distribution by applying Maximum Likelihood Method. Let $f(t, a_{ij}, b_{ij})$ be density function of the time spent in state s_i and transition to state s_j. Based on transition probability matrix P and formula (3) then the density function of τ random variable denoting the sojourn time in state s_i (X random variable denoting the state of the system takes a value s_i) is as follows

$$f_i(t) = \sum_{j=1}^{k} f(t, a_{ij}, b_{ij}) \hat{p}_{ij} \tag{5}$$

but from (4), the distribution function of the sojourn time in state s_i is given as follows

$$F_i(t) = P(\tau \leq t | X = s_i) = \sum_{j=1}^{k} \hat{p}_{ij} \int_0^t f(z, a_{ij}, b_{ij}) dz \tag{6}$$

where $1 \leq i \leq k$.

3.3. States detection and simulation of cycles

States detection

The vehicle's passage in real conditions is described by staying in the states A − acceleration, B − braking, D − driving, S − stop. The sequence of states creates cycles. To identify the states in real conditions, the authors analysed the vehicle speed reading (km/h), which was presented as a time series $\{v_t\}_{0\leq t\leq n}$, where $v_0 = 0$. For each moment, $1 \leq t \leq n$ was considered the subsequence $\{v_t\}_{\max(0,t-p)\leq t\leq \min(n,t+p)}$, where $p \geq 1$, $p \in \mathbb{N}$ denotes the maximum displacement from the moment t for which the speeds are taken into account. The analysis of the subsequence $\{v_t\}_{\max(0,t-p)\leq t\leq \min(n,t+p)}$ (containing at most $2p + 1$ realisations) allows us to eliminate instantaneous changes in the vehicle's state. To identify the vehicle's state at time t based on the realisation $\{v_t\}_{\max(0,t-p)\leq t\leq \min(n,t+p)}$ the linear dependence of velocity on time was analysed, and the authors considered a model

$$v_j = \alpha_0^t + \alpha_1^t (j - t) + \varepsilon \tag{7}$$

where ε is a random variable with a normal distribution $N(0, \sigma^2)$ and $\max(0, t - p) \leq j \leq \min(n, t + p)$.

Using the least squares method [10], the structural parameters of the model (7) were determined and the sequences $\{\alpha_0^t\}_{0\leq t\leq n}$ and $\{\alpha_1^t\}_{0\leq t\leq n}$ were obtained, where for the moment $t = 0$ was assumed $\alpha_0^0 = \alpha_1^0 = 0$. The parameter estimator α_0^t denotes the average velocity in the time interval from the moment $\max(0, t - p)$ to the moment $\min(n, t + p)$, while α_1^t denotes the acceleration in this interval. To classify the states, v_{min} (minimum vehicle speed below which we assume that the car is stopped) and α_{min} (absolute value of acceleration below the level α_{min} means driving) were established, constant or stop). Based on the series of vehicle speed readings $\{v_t\}_{0\leq t\leq n}$ the states were defined as follows

$$x_t = \begin{cases} S, & \alpha_0^t < v_{min} \quad \text{or} \quad v_t = 0 \\ A, & \alpha_0^t \geq v_{min} \quad \text{and} \quad \alpha_1^t > \alpha_{min} \\ B, & \alpha_0^t \geq v_{min} \quad \text{and} \quad \alpha_1^t < -\alpha_{min} \\ D, & \text{other} \end{cases} \tag{8}$$

Acceleration estimation

While moving the vehicle, changes in speed were also analysed in states A − acceleration, B − braking, D − driving. For each of the above mentioned states the acceleration was modelled using a normal distribution $N(m, \sigma^2)$, where the density function is defined as follows

$$f(x, m, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{9}$$

where $m \in \mathbb{R}$ and $\sigma > 0$. The parameters m (mean) and σ (standard deviation) were estimated by applying the Maximum Likelihood Method [19].

3.4. Cycle simulation algorithm

Below, the algorithm (Fig. 4) of cycle simulation is presented. According to the semi-Markov process [7], this algorithm consists of two parts. The first part contains a simulation of the sequence of vehicle states which has Markov property (1), but in the second part, the sequence of sojourn times with subsequence's of vehicle speed according to formula (8) is simulated.

1. Put $v_0 = 0$, $t = 0$ and vehicle state $x_0 = S$
2. Draw a sequence of states $\{x_t\}_{1 \le t \le n}$, whose realisations correspond to a Markov process with a probability matrix P and $x_t \in \{A, B, D, S\}$
3. Choice $\{x_t\}_{1 \le t \le m}$ realisation, where $m = \max\{k: x_k = S, 1 \le k \le n\}$ (truncate the sequence, where the last state is Stop)
4. Put $j = 0$, $T_0 = 0$
5. While $j \le m$

$j = j + 1$

case: $x_j = S$

draw τ_j with $F_S(t)$ distribution function (6)

for $i = 1: \lceil \tau_j \rceil$:

$t = t + 1$

$v_t = 0$

$T_j = t$

case: $x_j = B$

draw τ_j with $F_B(t)$ distribution function (6)

while $v_t > 0$:

$t = t + 1$

draw g with $N(m_B, \sigma_B^2)$ distribution (draw acceleration for braking state)

$$v_t = \begin{cases} \max(0, v_{t-1} + g), & t \le T_j + \tau_j \\ 0, & t > T_j + \tau_j \end{cases}$$

$T_j = t$

case: $x_j = A$

draw τ_j with $F_A(t)$ distribution function (6)

while $v_t < v_{max}$ and $t < T_j + \tau_j$:

$t = t + 1$

draw g with $N(m_A, \sigma_A^2)$ distribution (draw acceleration for acceleration state)

$v_t = \min(v_{max}, v_{t-1} + g)$

$T_j = t$

case: $x_j = D$

draw τ_j with $F_D(t)$ distribution function (6)

while $t < T_j + \tau_j$:

$t = t + 1$

draw g with $N(m_D, \sigma_D^2)$ distribution (draw acceleration for driving state)

$v_t = \max\left(0, \min(v_{max}, v_{t-1} + g)\right)$

$T_j = t$

As a result of the algorithm, we obtain $\{x_j\}_{0 \le j \le m}$ sequence of states and $\{T_j\}_{0 \le j \le m}$ the sequence of moments, where the vehicle state is changed, and $\{v_t\}_{0 \le j \le T_m}$ sequence of vehicle speed.

3.5. Road emissions

For the pollutant, the data set $D = \{(v_t, b_t): 1 \le t \le n, v_t \ge 2\}$ was analysed, where v_t denotes the vehicle speed at the time t, b_t – the pollutant at that time, $1 \le t \le n$. In state S (stop, vehicle speed less than v_{min}) the expected emission $\hat{b}_S$ for each moment is estimated as the mean of pollutant when the vehicle had state S

$$\hat{b}_S = \frac{1}{n_S} \sum_{i \in \{k: x_l = S, 1 \le l \le n\}} b_i \tag{10}$$

where $n_S = \#\{k: x_l = S, 1 \le l \le n\}$ denotes the number of moments where the vehicle was in state S.

Readings where the vehicle speed exceeds v_{min} (vehicle takes the states: Braking, Acceleration, Driving) were used for the analysis. From the Sensors Inc. analyser, type Semtech DS, pollutants in g/s were recorded. Then, for each pollutant for the time t, we determine the road emission road emission expressed in [g/km] as follows

$$y_t = \frac{b_t}{v_t} 3600$$

for $1 \le t \le n$.

The authors analysed the dependence of road emissions y on vehicle speed v, i.e. $y_t = y(v_t)$. The relationship between emissions and speed was presented as follows

$$y = \alpha_0 + \alpha_1 v + \alpha_2 \left(\frac{v}{\alpha_3}\right)^{\alpha_4 - 1} e^{-\left(\frac{v}{\alpha_3}\right)^{\alpha_4}} + \varepsilon \tag{11}$$

where ε is a normally distributed random variable $N(0, \sigma^2)$. In the equation (11) the component $\alpha_0 + \alpha_1 v$ represents the scale of pollutant emissions associated with higher vehicle speed (it has a smaller effect at lower speeds), while the component $\alpha_2 \left(\frac{v}{\alpha_3}\right)^{\alpha_4 - 1} e^{-\left(\frac{v}{\alpha_3}\right)^{\alpha_4}}$ represents the scale of pollutant emissions associated with lower vehicle speed (it has a smaller effect at higher speeds). The parameter α_2 is responsible for the scale of pollutants generated by the vehicle for higher speeds, and the parameters α_3 i α_3 correspond to the scale and shape of the impact of lower speeds, on the pollutant emission curve (they are mainly responsible for the rate of emission decrease with increasing speed). According to the information presented below, we have an exponential decrease in pollution for lower speeds. In comparison, for higher speeds, we have an increase similar to a linear trend (from (11)). It is not exactly a linear dependence on speed, but as speed increases with the appropriate choice of parameters α_3 and α_4 in the limit, we have a linear trend).

Based on the sequence $\{(y_t, v_t)\}_{1 \le t \le n}$ we estimate the unknown parameters in equation (11) by using the least squares method [18, 19] and solving the optimisation problem

$$\min_{\alpha \in G} F(\alpha) \tag{12}$$

where the objective function is given as follows

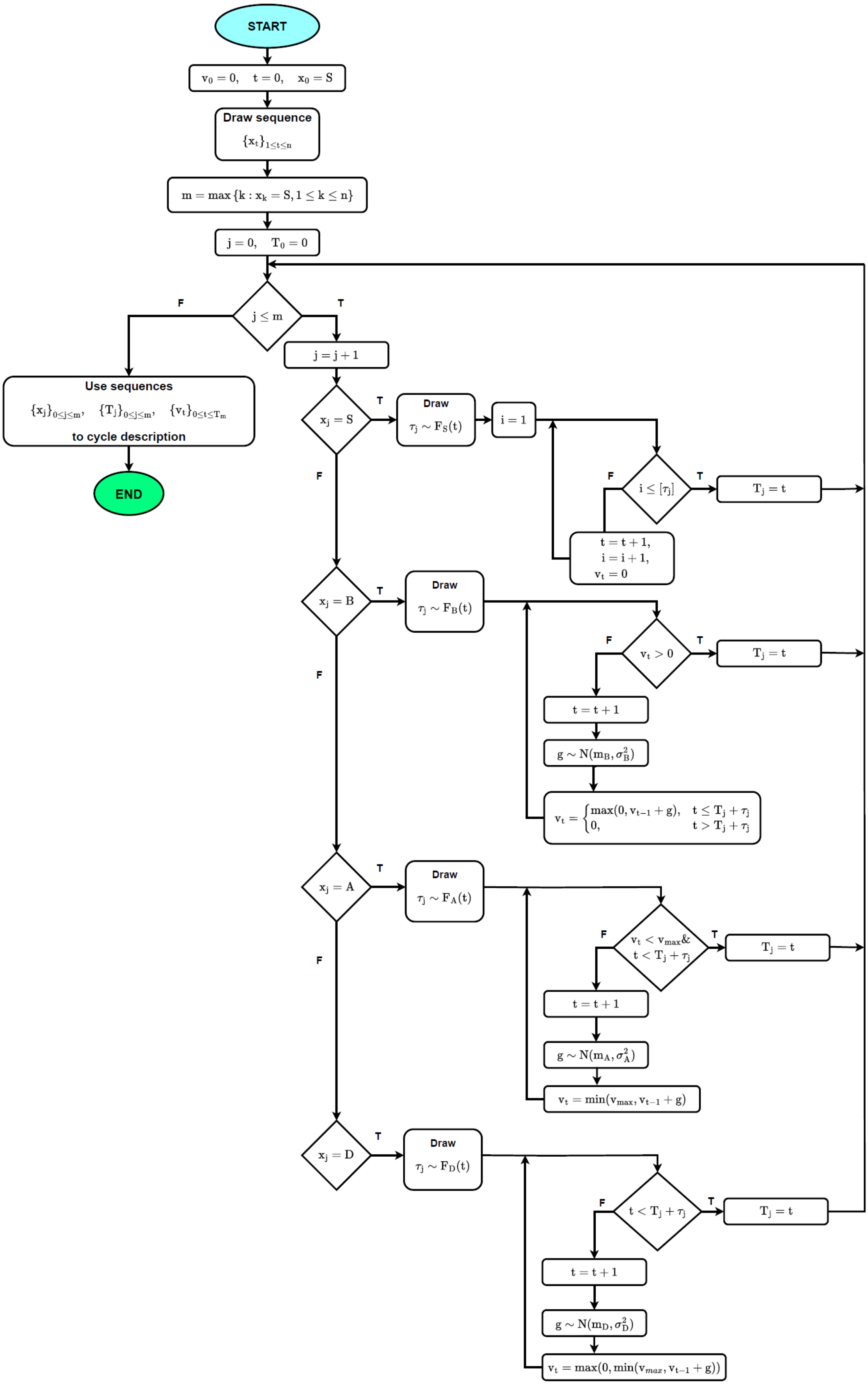

Fig. 4. Diagram of the cycle simulation algorithm

$$F(\alpha) = \sum_{t=1}^{n} \left(y_t - \alpha_0 - \alpha_1 v_t - \alpha_2 \left(\frac{v_t}{\alpha_3}\right)^{\alpha_4 - 1} e^{-\left(\frac{v_t}{\alpha_3}\right)^{\alpha_4}} \right)^2 \quad (13)$$

while the set of possible solutions is defined as $G = \{(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4) \in \mathbb{R}^5 : \alpha_0, \alpha_1, \alpha_2, \alpha_3, \geq 0, 0 < \alpha_4 < 1\}$.

4. Results and discussion

Figure 5 shows the vehicle course determined using the described model, with the individual states marked braking, acceleration, driving, and stop.

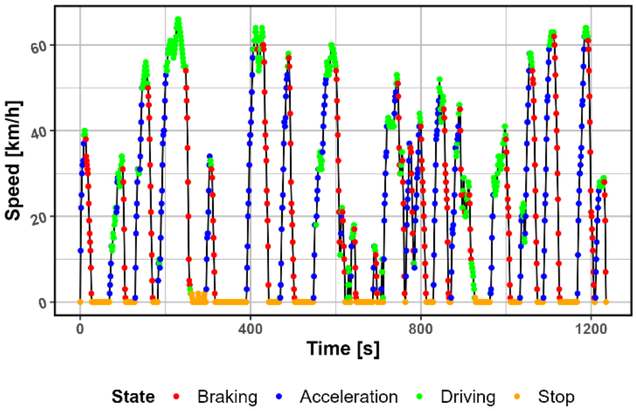

Fig. 5. Vehicle mileage determined using the model

The probabilities of transitions between states were estimated (Table 1). The table below shows the transition probability matrix between states, while Fig. 6 shows the transition probability between states.

Table 1. State transition probability matrix

	Braking	Acceleration	Driving	Stop
Braking	0.00	0.000	0.320	0.680
Acceleration	0.00	0.000	1.000	0.000
Driving	0.61	0.293	0.000	0.098
Stop	0.00	0.857	0.143	0.000

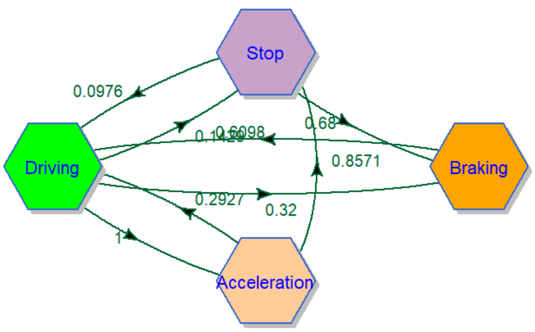

Fig. 6. Transition probability between states

A parametric identification of the Weibull distribution was performed for random variables describing the transition time between states (Tables 2 and 3).

Table 2. Shape parameters of Weibull distribution of transition time between states

	Braking	Acceleration	Driving	Stop
Braking	–	–	1.956	3.482
Acceleration	–	–	1.807	–
Driving	1.173	1.065	–	2.617
Stop	–	0.915	1.854	–

Table 3. Scale parameters of Weibull distribution of transition time between states

	Braking	Acceleration	Driving	Stop
Braking	–	–	6.454	11.729
Acceleration	–	–	9.121	–
Driving	10.408	4.966	–	6.222
Stop	–	20.467	25.055	–

The examination of transition durations between vehicle states offers valuable information into vehicle movement across various driving stages. Transitions from braking to driving are typically rapid and very predictable, indicating that braking frequently functions as a brief adjustment prior to the vehicle returning to stable motion. Conversely, transitioning from braking to a complete stop is a more gradual and consistent procedure, typically requiring a longer duration, which is logical considering the necessity to halt the vehicle smoothly.

The data indicates that the transition to driving acceleration exhibits variability, since the vehicle requires time to stabilise its speed following an increase in velocity. Transitions from driving to braking are similarly affected by several circumstances, including road conditions and traffic, resulting in increased variability, however they transpire rather swiftly. Transitions from driving to a stop are notably more direct and expedient, perhaps because halting frequently occurs in response to traffic signals, junctions, or other unequivocal indicators.

Using the formula (5), the density functions were estimated as a mixture of Weibull densities for sojourn time for each state. The results are shown in Fig. 7. The figure offers a detailed examination of the duration vehicles generally spend in various states – braking, accelerating, driving, and stopping – and illustrates distinct behavioral tendencies. Braking and accelerating are rapid and transient, whereas driving represents the primary, stable condition. Stopping exhibits significant variability, encompassing brief pauses to extended delays. These patterns illustrate the inherent cadence of driving and halting, presenting opportunities to enhance driving fluidity and efficiency. For instance, reducing superfluous stops or enhancing transitions between states may conserve fuel and diminish emissions. This analysis can enhance traffic flow, vehicle performance, and the overall driving experience.

For example, based on model (11), structural parameters of road emissions were estimated for nitric oxide (NO) and methane (CH_4) pollutants, as pollutants that are most sensitive to changes in vehicle speed, especially in urban traffic, where changes in speed lead to their increase [18].

The structural parameters for the above pollutants are estimated by solving task (12) and presented in Table 4, while the model's fit (11) to empirical data is presented in Fig. 8 and 9.

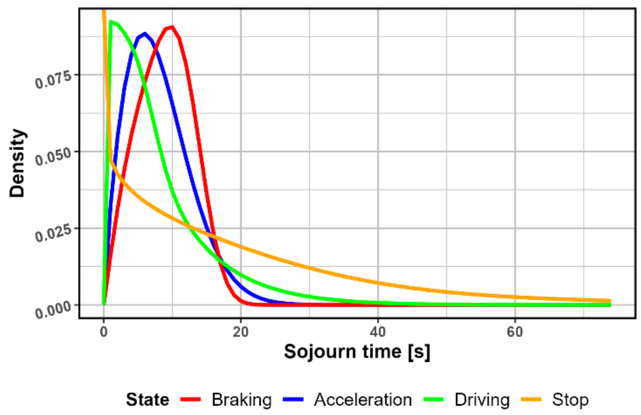

Fig. 7. Density functions of sojourn time in states

Figure 7 highlights the effects of vehicle speed on nitric oxide emissions. At very low speeds – below 10 km/h – emissions are higher, which makes sense given that stop-and-go traffic or idling in city environments often leads to inefficient fuel use and higher pollution.

Emissions drop and reach their lowest point around 20–25 km/h as the car accelerates. With cleaner and more consistent combustion, this suggests that engines work most effectively at these modest speeds.

The trend changes and nitric oxide emissions begin to grow continuously at speeds above 30 km/h, though. Greater engine loads, more fuel consumption, and higher combustion temperatures are probably the causes of this rise. Emissions vary greatly at speeds above 50 km/h, which could be affected by things like road inclines, vehicle condition, or driver acceleration and maintenance of speed.

Table 4. Structural parameter values and standard deviation of residuals

	α_0	α_1	α_2	α_3	α_4
NO	0	0.0264316	4.3705415	2.198382	0.8459142
CH_4	0.0138863	0.0023000	0.3217594	14.765735	0.0039383

Figure 8 reveals some fascinating trends and shows unequivocally how vehicle speed influences methane emissions. Emissions are rather high, usually exceeding 1.0 units, at extremely low speeds – less than 10 km/h. This makes it logical, as idling in cities or stopping-and-go traffic usually results in poor fuel economy and incomplete combustion. Emissions drop significantly, though, and stabilise at about 20 km/h as the car accelerates. This reveals that engines generate less methane and run far more effectively at reasonable speeds.

Methane emissions remain low and consistent between 20 and 40 km/h, indicating that this speed range is excellent for the lowest emissions. Emissions start to rise somewhat after 40 km/h, probably because faster speeds demand more from the engine, therefore increasing fuel consumption and lowering combustion efficiency. Still, this rise is far smaller than the sharp decline experienced at slower speeds.

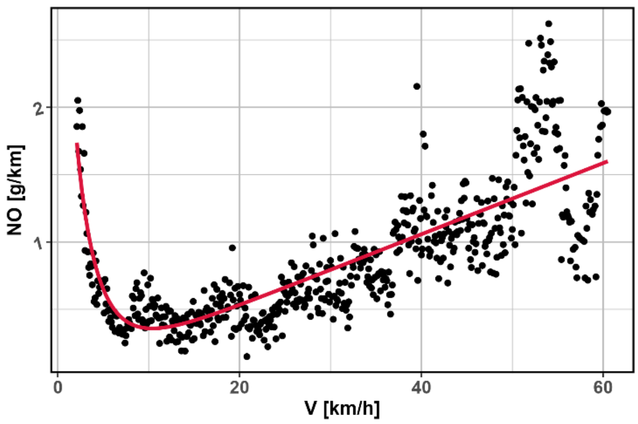

Fig. 8. Dependence nitric oxide (NO) emission of road nitrogen oxide emissions on speed

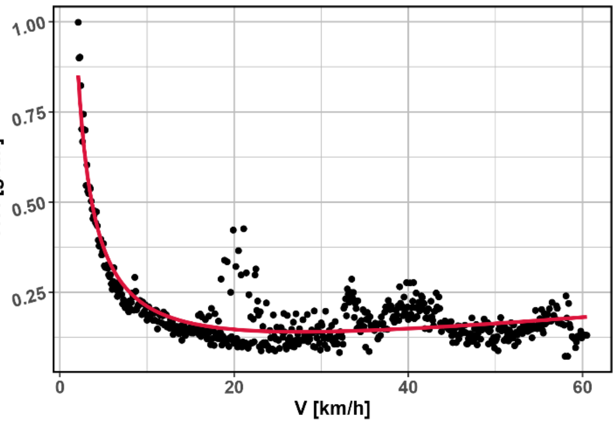

Fig. 9. Dependence methane (CH_4) emission of road methane emissions on speed

The data also shows variation in emissions, especially at slower speeds, which could rely on elements including the type of vehicle, its condition, fuel quality, or driving style. This unpredictability reduces as speed rises, most likely when the engine runs more steadily.

Overall, engines are most effective between 20 and 40 km/h, the sweet spot for low methane emissions. High emissions at slower speeds draw attention to the need for improved urban traffic management – minimising idling and maximising stop-and-go strategies. The little increase at higher speeds indicates, nevertheless, the need to drive effectively on open highways to control emissions. These realisations can support better, more environmentally friendly driving habits.

From formulas (10) and (11) based on time series $\{v_t\}_{1\le t\le p}$ denoting the speed of a vehicle and corresponding them the sequence of states$\{x_t\}_{1\le t\le p}$ we estimate the expected total pollution during vehicle mileage as follows

$$B = n_S \hat{b}_S + \sum_{i\in\{k:x_l\neq S,1\le l\le p\}} y\,(v_i)\,\frac{v_i}{3600} \tag{14}$$

where $n_S = \#\{k: x_l = S, 1 \le l \le p\}$.

For vehicle mileage presented in Fig. 5 and dependence of road emissions on speed presented in Fig. 8 and 9, the expected total nitrogen oxide pollution equals 1.119 g and methane pollution equals 0.641 g.

5. Conclusions

The study employed a semi-Markov process to identify the actual vehicle journey, with transition probabilities between states and idle times within states modelled using a Weibull distribution. This approach captured changes in speed during braking, acceleration, and driving states. Leveraging these elements, an algorithm was developed to simulate driving cycles, accurately reflecting the vehicle's mileage.

This versatile algorithm enables simulations of various travel durations; while the study demonstrates a 20-minute drive, the algorithm can accommodate simulations of any desired driving time. A model was used to estimate road emissions, which correlates emissions with vehicle speed and is characterised by an exponential decrease at lower speeds and a distinct trend at higher speeds.

Integrating the algorithm and the emissions model allows vehicle journeys to be simulated under real traffic conditions. This integration facilitates the estimation of expected emissions across the entire speed profile, accounting for the nuances of actual driving scenarios.

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