Article citation info:
Żmudka Z. Improving the uncertainty of measured and calculated data in engine problems using the equalisation calculus. Combustion Engines. 2026;204(1):153-157. https://doi.org/10.19206/CE-210987

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Combustion Engines
Polish Scientific Society of Combustion Engines

Improving the uncertainty of measured and calculated data in engine problems using the equalisation calculus

ARTICLE INFO

Received: 26 May 2025
Revised: 11 August 2025
Accepted: 16 September 2025
Available online: 26 September 2025

A selected example of the application of the equalization calculus to reconcile the results of experimental tests of a spark-ignition internal combustion engine is presented. The example concerns the selection of characteristic parameters of the theoretical Seiliger-Sabathe cycle in accordance with the experimentally determined real cycle. This is therefore a so-called inverse problem. The isochoric and isobaric loading parameters and the heat distribution number were reconciled. In addition, the effect of applying the reconciliation algorithm twice on the correction of measurement results with gross errors is presented.

Key words: *spark ignition engine, theoretical cycle, real cycle, measurement deviation, equalisation calculus*

1. The essence of using the equalisation calculus

The equalisation calculus is used in many fields of engineering sciences and industry where experimental research is carried out. It is a practical tool for the authentication and validation of measurement results [1, 5, 9, 11, 12].

In experimental testing of internal combustion engines, there are often situations where the number of calculated unknown quantities is less than the number of independent equations relating these quantities to the measured data. The unknowns determined directly from these equations can be calculated in a number of ways, depending on the choice of equation set. The remaining dependencies will not be fulfilled exactly, which is due to the occurrence of unavoidable measurement errors, and consequently, the same quantities will take different values. In order to avoid these differences (obtaining one parameter value) and to get the consistency of all equations, it is necessary to carry out a procedure to reconcile the equations by means of the methods of equalization calculus [8, 13]. The essence of this calculus is the correction of measurement results, after which the results of calculations of unknown quantities, determined from different sets of equations, will be the same.

The classic situation where the number of equations is greater than the number of unknowns also often occurs when carrying out measurements of the composition of exhaust gases, aimed at determining those quantities that are difficult or impossible to measure directly. The calculation algorithm then uses the balance equations of the elements involved in the combustion process. The unknowns (unit amount of air n'_a and exhaust gas n''_{ss}) can be calculated in several ways, depending on the choice of the set of balance equations. The influence of measurement errors on the deviations of the calculation results should be decisive in this respect. However, in this article, a different example will be presented, involving the application of the equalisation calculus to determine the characteristic parameters of the theoretical Seiliger-Sabathe cycle.

The subject of the article concerns primarily the problem of applying the equalization calculus in engine problems. The problem of adjusting the Seiliger-Sabathe cycle to a real cycle served only as an example of its application.

Generally, all independent equations (so-called equations of conditions) form a system of functions of the general form [8, 10]:

$$F_k = F_k\left(x_1,\dots, x_i,\dots, x_n, y_1,\dots, y_j,\dots, y_u\right), \quad k = 1\dots r \tag{1}$$

where: x_i – measured quantity (i = 1 ... n), n – number of measured quantities, y_j – unknown quantity (j = 1 ... u), u – number of unknown quantities, r – number of equations of the conditions.

The equations of conditions forming the system (1) must satisfy the mutual independence condition and the determinability condition for the unknowns. The verification of these conditions involves examining the order of the appropriate Jacobi matrices from the partial derivatives of the function F_k according to the arguments x_i and y_j [8].

By substituting the results of the measurements and the approximate (pre-calculated) values of the unknowns into the equations of the conditions, some of them do not meet. A system of equations is obtained:

$$F_k\left(x_{1,0} \dots x_{i,0} \dots x_{n,0}\ ,\ y_{1,0} \dots y_{j,0} \dots y_{u,0}\right) = -w_k \tag{2}$$

where: $x_{i,0}$ – result of measurement of the i-th quantity (i = 1 ... n), $y_{j,0}$ – approximate value of the j-th unknown (j = 1 ... u), w_k – incompatibility of the k-th equation of condition (k = 1 ... r).

Obviously, the incompatibilities w_k of the equations of the conditions used for the initial calculation of the unknowns are zero ($w_k = 0$).

In order to obtain compatibility of all the equations of the conditions, it is necessary to introduce corrections ν_i to the measurement results and corrections δ_j to the approximate values of the unknowns. These corrections are calculated from the system of equations [10]:

$$\begin{cases} \sum_{i=1}^{n} a_{ki}\, v_i + \sum_{j=1}^{u} b_{kj}\, \delta_j = w_k & (k = 1 \dots r) \\ \dfrac{v_i}{m_i^2} = \sum_{k=1}^{r} a_{ki}\, k_k & (i = 1 \dots n) \\ \sum_{k=1}^{r} b_{kj}\, k_k = 0 & (j = 1 \dots u) \end{cases} \tag{3}$$

where: k_k – correlates, coefficients in determining conditional extreme, m_i – mean absolute error of the measurement result of the i-th quantity.

The coefficients a_{ki} and b_{kj} of the above system of equations are the partial derivatives of the function F_k according to the measured quantities x_i and according to the unknowns y_j, respectively, calculated at a point with coordinates:

$$\left(x_{1,0} \dots x_{i,0} \dots x_{n,0}\ ,\ y_{1,0} \dots y_{j,0} \dots y_{u,0}\right).$$

They are therefore calculated from the relationship:

$$a_{ki} = \left(\frac{\partial F_k}{\partial x_i}\right)_0 \quad , \quad b_{kj} = \left(\frac{\partial F_k}{\partial y_j}\right)_0 \tag{4}$$

2. Reconciling the parameters of the Seiliger-Sabathe cycle

The presented example concerns the problem of selecting the parameters of the theoretical Seiliger-Sabathe cycle (Fig. 1) according to the experimentally determined real engine cycle [2, 14]. This is therefore the so-called inverse problem.

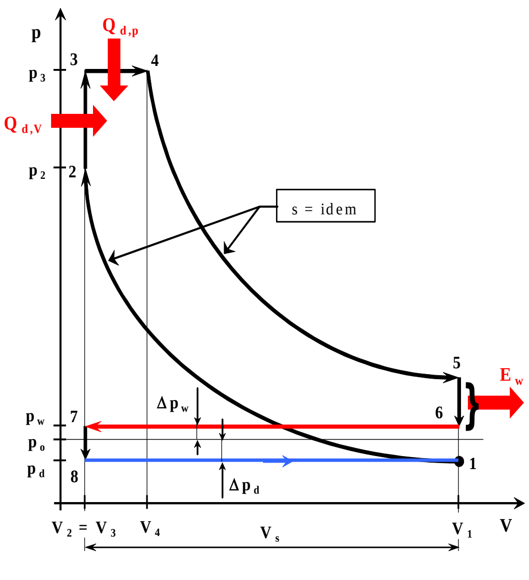

Fig. 1. Open Seiliger-Sabathe cycle

In order to unambiguously determine the course of the Seiliger-Sabathe cycle, it is necessary to determine [14]:

– isochoric load parameter (pressure increase ratio) γ:

$$\gamma = \frac{p_3}{p_2} \quad , \quad \gamma \geq 1 \tag{5}$$

– and isobaric load parameter (volume increase ratio) φ:

$$\varphi = \frac{V_4}{V_3} \quad , \quad \varphi \geq 1 \tag{6}$$

from here, the heat distribution number Ψ can be determined, defined as:

$$\Psi = \frac{Q_{d,v}}{Q_d} \quad , \quad 0 \leq \Psi \leq 1 \tag{7}$$

The quantities used to define the above parameters are marked in Fig. 1, which shows the Seiliger-Sabathe cycle in the p-V system. Defined parameters (5), (6) and (7) are linked together by independent formulas:

$$\gamma = 1 + \frac{E\, \Psi\, (\kappa - 1)}{\varepsilon^{(\kappa-1)}} \tag{8.1}$$

$$\varphi = 1 + \frac{(\kappa - 1)\, E\, (1 - \Psi)}{\kappa \left(E\, \Psi\, (\kappa - 1) + \varepsilon^{(\kappa-1)}\right)} \tag{8.2}$$

where: $\varepsilon = V_1/V_2$ – compression ratio, E – energy-stoichiometric parameter, defined as [7]:

$$E = \frac{Q_d}{p_1 V_1} \tag{9}$$

In the case of experimental (based on indirection) determination of approximate, preliminary values of the parameters γ_0 and φ_0, as well as knowledge of parameter E and compression ratio ε, there will be one unknown in two equations (8) – the heat distribution number Ψ. Thus, in this situation, it is possible to use the equalization calculus to obtain the reconciliation of the relations (8), which are the equations of the conditions.

The following designations have been adopted:

γ_0 – experimentally determined isochoric load parameter (preliminary value)

φ_0 – experimentally determined isobaric load parameter (preliminary value)

Ψ_0 – pre-calculated value of the heat distribution number from the formula (8.1)

v_γ – correction of the isochoric load parameter γ

v_φ – correction of the isobaric load parameter φ

δ – correction of the heat distribution number Ψ

$m_\gamma = m_\varphi = 0.1$ – mean absolute error of the measurement result γ and φ.

Taking the first of the equations of the conditions (8.1) for the initial calculation of the unknown heat distribution number Ψ, the system of equations (3) allowing the calculation of the corrections v_γ, v_φ, and δ will take the form (10):

$$-\varepsilon^{(\kappa-1)} v_\gamma + (\kappa - 1) E\, \delta = 0 \tag{10.1}$$

$$\left[\kappa\, E\, (\kappa - 1)\Psi_0 + \kappa\, \varepsilon^{(\kappa-1)}\right] v_\varphi + (\kappa - 1) E \left[(\varphi_0 - 1)\kappa + 1\right]\delta = w_2 \tag{10.2}$$

$$\frac{v_\gamma}{m_\gamma^2} = -\varepsilon^{(\kappa-1)} k_1 \tag{10.3}$$

$$\frac{\nu_{\varphi}}{m_{\varphi}^{2}} = \left[\kappa\, E\,(\kappa - 1)\,\Psi_0 + \kappa\,\varepsilon^{(\kappa-1)}\right] k_2 \qquad (10.4)$$

$$(\kappa - 1)\,E\,k_1 + \left[(\varphi_0 - 1)\,\kappa\,E\,(\kappa - 1) + (\kappa - 1)\,E\right] k_2 = 0 \qquad (10.5)$$

Once the corrections have been calculated, the final corrected values can be determined:
– isochoric load parameter:

$$\gamma_u = \gamma_0 + \nu_\gamma \qquad (11)$$

– isobaric load parameter:

$$\varphi_u = \varphi_0 + \nu_\varphi \qquad (12)$$

– and also, the number of heat distribution:

$$\Psi_u = \Psi_0 + \delta \qquad (13)$$

The effect of reconciliation for the analysed example is presented graphically in Fig. 2.

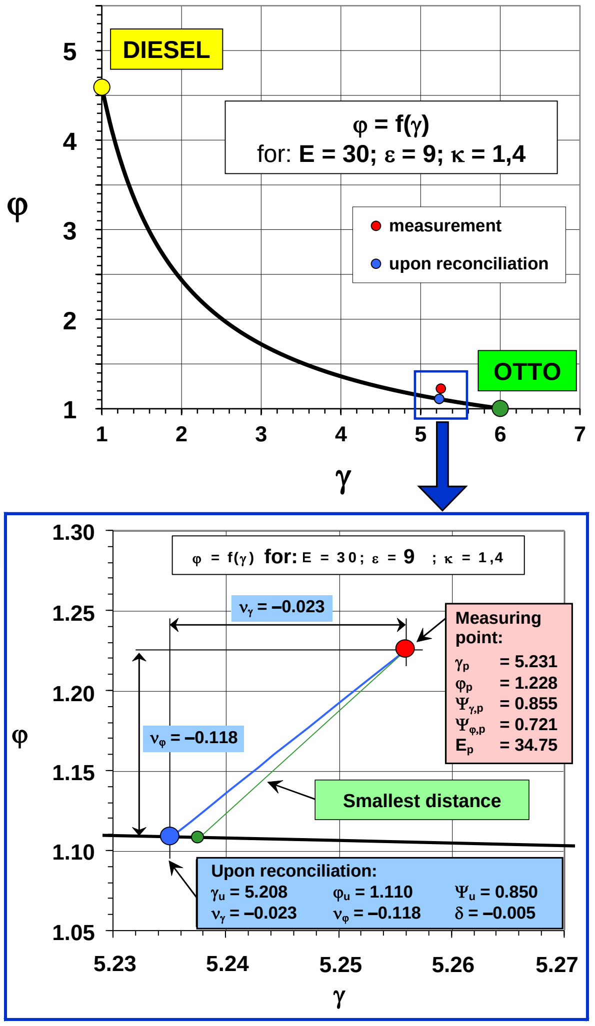

Fig. 2. Results of parameter (γ and φ) reconciliation for the Seiliger-Sabathe cycle

The experimentally determined measuring point does not lie on the characteristic showing the exact course of the relationship $\varphi = f(\gamma)$ due to measurement errors. Also, for this reason, two different values of the unknown heat distribution number ($\Psi_{\gamma,p}$ and $\Psi_{\varphi,p}$ – Fig. 2) were obtained from the two relations (8.1) and (8.2). After applying the reconciliation procedure of the formulas (8), there was a shift of the measurement point to the line $\varphi = f(\gamma)$, because now the relations (8) are strictly satisfied. Both formulas (8) also give the same values of the heat distribution number (marked Ψ_u in Figure 2). It is characteristic that the reconciliation algorithm does not move the measurement point to the line $\varphi = f(\gamma)$ according to the smallest distance (Fig. 2).

The corrections ν_γ and ν_φ (Fig. 2) satisfy the condition:

$$|\nu_i| < 3|m_i| \qquad (14)$$

which satisfies the conditions for the applicability of the reconciliation method [9]. Therefore, the assumed accuracy of the measurements was maintained, and the reconciliation results can be considered satisfactory. At engine operating points where the above condition (14) is not met, the measurement results should be rejected.

Figure 3 shows, for illustration, the effect of double reconciling measurements with gross errors [4, 6]. The measuring point is located at a greater distance from the exact course of the relationship $\varphi = f(\gamma)$. In such a situation, the reconciliation procedure does not cause this point to be shifted to the line $\varphi = f(\gamma)$. Only the repeated application of the reconciliation algorithm shifts the point to the line of the exact function $\varphi = f(\gamma)$. However, in this case, the values of the corrections ν_γ and ν_φ do not satisfy condition (14) for the applicability of the reconciliation algorithm.

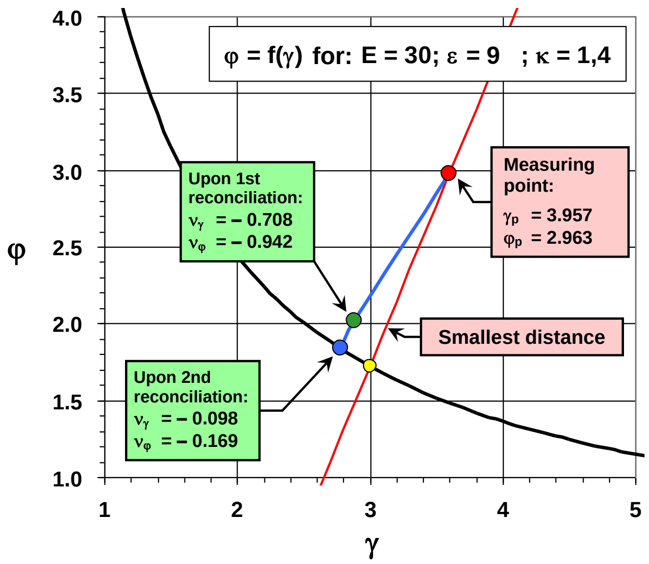

Fig. 3. Effect of double reconciliation on the correction of measurement results with gross errors

This means that the accuracy of the measurements has not been met. In this situation, the measurement results should be discarded because the errors exceed the required range. It is advisable to repeat the measurement of the parameters γ and φ for this engine operating point.

The reconciliation of measured quantities enables the determination of unambiguous and most probable values of unknowns along with an assessment of their accuracy. The errors m_j of the unknowns y_j after reconciliation can be calculated according to the law of error transfer according to the relationship [7, 8]:

$$m_j^2 = \left(\frac{\partial y_j}{\partial x_1}\right)_0^2 m_1^2 + \ldots + \left(\frac{\partial y_j}{\partial x_i}\right)_0^2 m_i^2 + \ldots + \left(\frac{\partial y_j}{\partial x_n}\right)_0^2 m_n^2 \quad (15)$$

The presented example of the problem of selecting the parameters of the theoretical Seiliger-Sabathe cycle (load parameters γ, φ, and heat distribution number Ψ as an unknown) with the use of the equalisation calculus fully confirmed the need for its use.

Using the presented algorithm for reconciling the mentioned parameters, the example effects of selecting the theoretical cycles according to the real cycles are then presented. Example experimental tests of the real cycles were carried out on a spark-ignition engine type 170A1.046, the basic data of which are presented in Table 1. A comparison of the real engine cycles with the corresponding Seiliger-Sabathe cycles for idle and full load is illustrated in Fig. 4 and Fig. 5, respectively.

Table. 1. Technical data of the tested SI combustion engine

Engine designation	170A1.046
Engine type	spark ignition, 4-stroke, naturally aspirated
Number and arrangement of cylinders	4-cylinders in line
Piston diameter and stroke	65 × 67.7 mm
Engine displacement	0.899 dm^3
Compression ratio	9

For both tested engine operating conditions, the degree of internal excellence was also calculated, which is defined as follows [7]:

$$\xi_i = \frac{L_i}{L_o} \quad (16)$$

where: L_i – internal work (work of the real cycle), L_o – work of the theoretical cycle, and its values are presented in Table 2.

Table. 2. Degree of internal excellence for tested SI engine loads

Engine load	ξ_i
Idling (Fig. 4)	0.250
Full load, 59.6 Nm/rad (Fig. 5)	0.584

A characteristic feature of the operation of a spark-ignition engine is a much lower value of the ξ_i degree at idle speed compared to higher loads, especially full load. One of the main reasons for this situation is the quantitative method of load regulation using a throttle valve as a flow throttling element. Thus, the degree of internal excellence ξ_i characterizes the quality of thermal and flow processes occurring during engine operation. Its value, and therefore also the internal work of the engine L_i (also called indicated work, i.e. work of real cycle), is significantly influenced by many factors, in particular:

– heat exchange (engine cooling)
– charge exchange work
– incomplete combustion
– the real composition of the combustible mixture
– fuel burnout course
– working medium as a real gas
– way and the ignition point of the mixture.

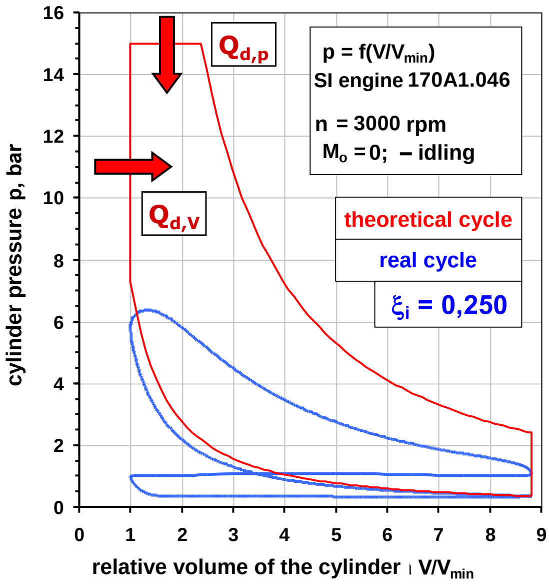

Fig. 4. Comparison of the real engine cycle with the corresponding theoretical Seiliger-Sabathe cycle for idling

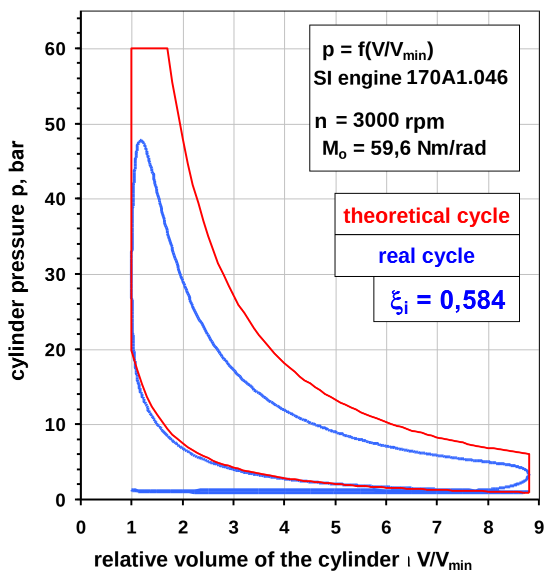

Fig. 5. Comparison of the real engine cycle with the corresponding theoretical Seiliger-Sabathe cycle for full load

The comparison of theoretical and real cycles presented in Fig. 4 and Fig. 5 additionally leads to the conclusion that the Seiliger-Sabathe cycle is also an appropriate theoretical cycle for the SI engine. Such a conclusion is justified by the fact that heat release during combustion in the spark-ignition engine also takes place at the beginning of the power stroke.

3. Conclusion

The essence, principles, and purpose of using the equalization calculus to reconcile measurement data are presented. The issue was illustrated by an example of reconciling the dependencies linking the parameters of the Seiliger-Sabathe theoretical cycle, which fully confirmed the need to use this calculus.

The reconciliation procedure is particularly useful in situations where the formulas from which the unknown quantities are calculated are particularly "sensitive" to measurement errors of experimentally determined quantities. The reconciliation algorithm provides the following advantages [3, 6, 8, 9]:

- unambiguous and most probable values of unknowns are obtained, along with an assessment of their accuracy
- probable errors of measurement results are reduced
- one obtains the ability to control whether the assumed accuracy of measurements has been met
- those measurement results whose probable error exceeded the required range can be rejected.

In addition, the equalization calculus can also be used to check the accuracy of quantities that are estimated, for example, on the basis of literature data and to control simplifying assumptions.

Acknowledgements

This research is supported by the Ministry of Education and Science (Poland) under statutory research funds of the Faculty of Energy and Environmental Engineering of Silesian University of Technology.

Nomenclature

E	energy-stoichiometric parameter
k	correlates
m	mean absolute error of the measurement result
M_o	torque
n	engine speed
p	pressure
Q	heat
s	specific entropy
V	volume
γ	isochoric load parameter (pressure increase ratio)
δ	correction of the heat distribution number
ε	compression ratio
ν_γ	correction of the isochoric load parameter
ν_φ	correction of the isobaric load parameter
φ	isobaric load parameter (volume increase ratio)
ξ_i	degree of internal excellence
Ψ	heat distribution number

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