

Modeling of stochastic passenger car velocity processes for the purpose of pollutant emissions inventory

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This paper discusses the potential use of stochastic processes as road vehicle velocity models for road transport emissions inventory purposes. Empirical studies have presented stochastic passenger-car velocity models, each modeling traffic conditions: in traffic congestion, in cities outside traffic congestion, outside cities, and on highways and expressways. Zero-dimensional characteristics of the model velocity processes have been examined. The characteristics of passenger car emissions for 2020 have been determined using simulation methods. Road pollutant emissions from passenger cars under specific velocity process implementations have been determined and analyzed. The research results have been assessed, among other things, for their variability. Based on the results, the feasibility of using stochastic processes as road vehicle velocity models for road transport emissions inventory purposes has been assessed.

Key words: road transport, inventory of pollutant emissions, vehicle velocity processes, stochastic processes

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1. Introduction

In the inventory of pollutant emissions, in accordance with the guidelines in force in the European Union [5–7], means of road transport are classified into categories based on:

- intended use
- conventional size
- combustion engine properties
- fuel
- technical level.

A category is a set of entities with specific characteristics and interconnected by relationships.

The elementary category of road vehicles is the set of vehicles that meet all the same criteria. The set of road vehicles that do not all share the same criteria is a cumulative category.

Cumulative vehicle categories based on their use are [5-7]:

- passenger cars – PC
- light commercial vehicles (light duty vehicles) – LCV
- heavy-duty trucks (heavy-duty vehicles) – HDT
- urban buses – UB
- long-distance buses (coaches) – C
- motorcycles, mopeds, quads, and microcars – L.

Fuel consumption and pollutant emissions from internal combustion engines depend on their operating states [4], including engine speed, load (most often measured by torque), and the engine's thermal state. The engine's thermal state can be described by the set of temperatures of engine parts and their operating fluids, primarily the engine oil and, in the case of indirect engine cooling, the coolant. With a stabilized engine thermal state, fuel consumption and pollutant emissions depend on engine speed and load. In static engine operating conditions, these are functional dependencies (functions with numerical values), while in dynamic conditions, they are operational dependencies (functional) [4].

For road vehicle engines, the engine operating states are determined by the vehicle velocity process. Therefore, pollu-

tant emissions and fuel consumption can be tested under vehicle traffic conditions (light vehicles: passenger cars, light duty vehicles, minibuses, and L-category vehicles) [15] or under conditions simulating vehicle traffic, as is the case with heavy vehicles (trucks and buses), where the tests involve the engine itself, in states simulating the engine states in vehicle traffic [6, 15, 16].

There are many publications presenting results of tests on pollutant emissions and fuel consumption from road vehicles (mainly light vehicles) under various conditions, as determined by different tests, including road conditions in the RDE (Real Driving Emissions) test [1, 6, 15].

In real driving conditions, there is a degree of uncertainty in engine operating states, primarily due to the uncertainty in the vehicle velocity process. This justifies treating the vehicle process as a stochastic process [11, 12]. A model of a stochastic process can be a set of its realizations [11, 12]. For this reason, this paper addresses problems related to research on fuel consumption and pollutant emissions under stochastic vehicle velocity processes.

2. Stochastic driving tests of a passenger car

The task of determining the realization of stochastic passenger car velocity processes was undertaken as part of our own work.

Figures 1–4 present four realizations of stochastic passenger car vehicle processes in traffic conditions [5, 7]: in traffic congestion – C, in cities outside traffic congestion – U, outside cities – R, and on highways and expressways – H.

The individual realizations were determined based on empirical studies of vehicle velocity in real driving conditions [1, 4]. For each traffic model, several dozen (more than 20) realizations of the velocity process were recorded at a frequency of 1 Hz. Then, based on the analysis of the zero-dimensional statistical characteristics of the recorded waveforms, four representative realizations of the modeled stochastic vehicle velocity processes were selected.

Each realization was processed using a Savitzky-Golay filter of second order [14] to reduce high-frequency noise in the signals [9, 10, 13].

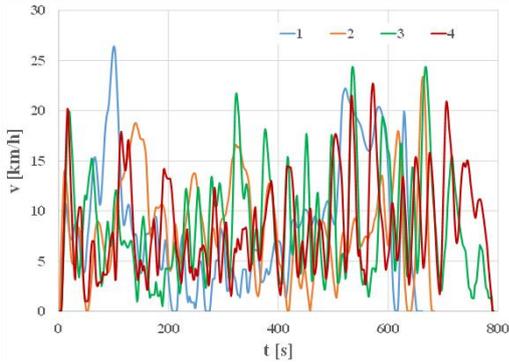


Fig. 1. Stochastic CT test: driving in cities in traffic congestion (own research)

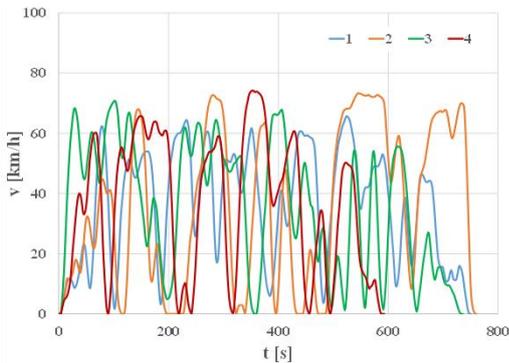


Fig. 2. Stochastic UT test: driving in cities outside traffic congestion (own research)

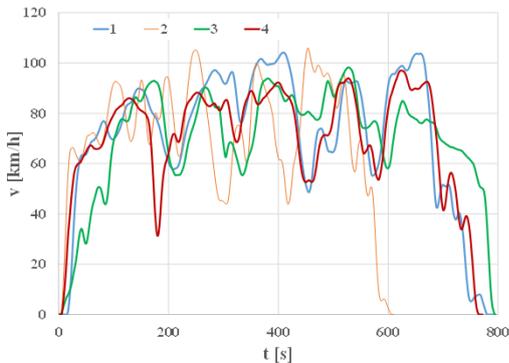


Fig. 3. Stochastic RT test: driving outside cities (own research)

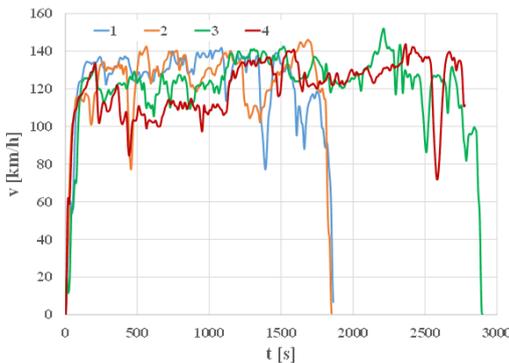


Fig. 4. Stochastic HT test: driving on highways and expressways (own research)

The individual implementations of the stochastic velocity models show many similarities, which will be quantitatively confirmed in the next chapter.

3. Driving test analysis

Table 1 presents the results of the statistical analysis of individual driving test implementations – zero-dimensional characteristics of individual implementations [9]:

Min – minimum value, Max – maximum value, Rg – range, AV – mean value, D – standard deviation, W – coefficient of variation.

Table 1. Zero-dimensional characteristics of driving tests

Test type	1	2	3	4	Min
	v _{AV} [km/h]				
CT	8.59	8.51	8.54	8.45	8.45
UT	36.04	36.00	35.98	36.02	35.98
RT	70.73	70.59	69.57	70.66	69.57
HT	122.49	122.99	121.38	117.94	117.94
Test type	Max	Rg	AV	D	W
	v _{AV} [km/h]				
CT	8.59	0.14	8.52	0.059	0.007
UT	36.04	0.06	36.01	0.025	0.001
RT	70.73	1.16	70.39	0.549	0.008
HT	122.99	5.05	121.20	2.276	0.019

Figure 5 shows the mean, minimum, and maximum values of the average velocity for individual model processes.

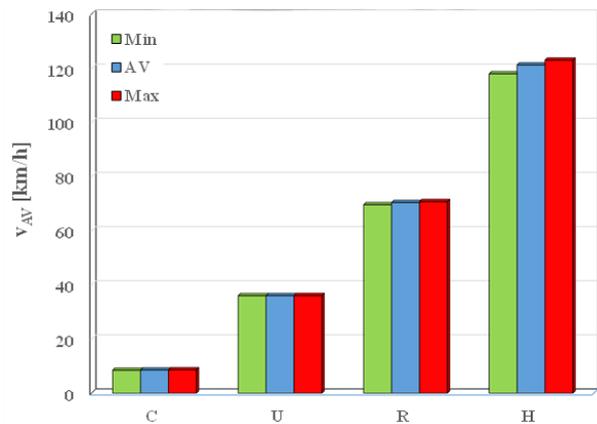


Fig. 5. Average value – AV, minimum value – Min and maximum value – Max of average velocity values – v_{AV} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

In urban traffic conditions, there is virtually no difference between the minimum and maximum average velocity values. The largest difference occurs for driving on motorways and expressways. Figure 6 presents the coefficient of variation of the velocity process for driving under model traffic conditions in individual implementations, as well as the average value of the coefficient of variation – AV.

The coefficient of variation is highest for the traffic model in traffic congestion, which results from the low average velocity – approximately 8.5 km/h. The coefficient of variation is lowest for driving on motorways and expressways – in this case, the average velocity is greater than

120 km/h. Figure 7 presents the coefficient of variation of the average velocity in individual process implementations under the model traffic conditions.

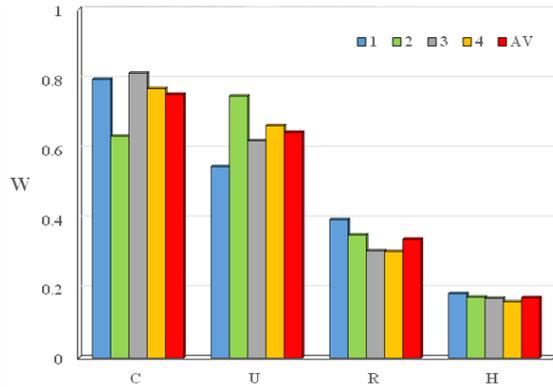


Fig. 6. Coefficient of variation of the velocity process for driving in model traffic conditions in individual realizations: in cities in traffic congestions – C, in cities outside traffic congestions – U, outside cities – R and on motorways and expressways – H and the average value of the coefficient of variation – AV

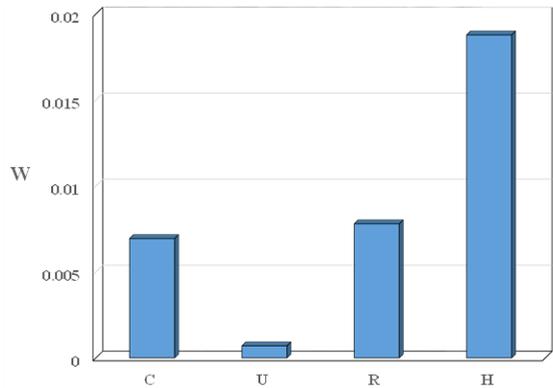


Fig. 7. Coefficient of variation of the average velocity value in individual process realizations in model traffic conditions: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

The greatest uniformity in average velocity across individual tests is observed for traffic in cities outside congestion, and the least for traffic on highways and expressways. Overall, the numerical values of the zero-dimensional characteristics of individual implementations indicate that implementations under the same model conditions exhibit significant similarity.

4. Sensitivity of fuel consumption and pollutant emissions to engine operating states in driving tests

To assess the sensitivity of fuel consumption and pollutant emissions to engine operating states during driving tests, test results for the passenger car category were obtained for 2020. Fuel consumption and pollutant emissions characteristics were determined using INFRAS software [8] for the state of engines heated to a stable temperature for all possible driving models. Average values were determined for mass fuel consumption (q), which is the derivative of the mass of fuel consumed relative to the distance traveled by the vehicle, and for road pollutant emissions (b), which

is the derivative of pollutant emissions relative to the distance traveled by the vehicle.

The pollutants considered were as follows: carbon monoxide (CO); hydrocarbons (HC); nitrogen oxides (NO_x); particulate matter (PM); and carbon dioxide (CO₂).

The sets of values of average mass road fuel consumption and road pollutant emissions as functions of average velocity were approximated by a fourth-degree polynomial function. This resulted in the characteristics shown in Fig. 8–13.

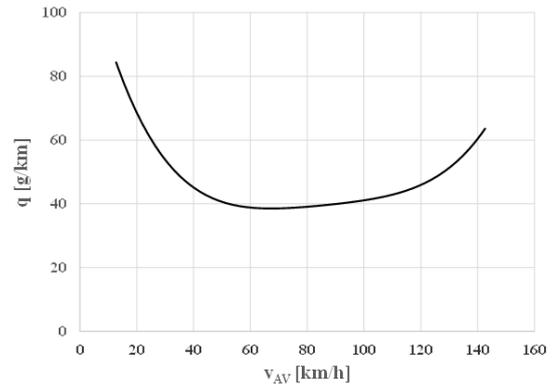


Fig. 8. Dependence of mass road fuel consumption on average velocity for the cumulative category of passenger cars in 2020

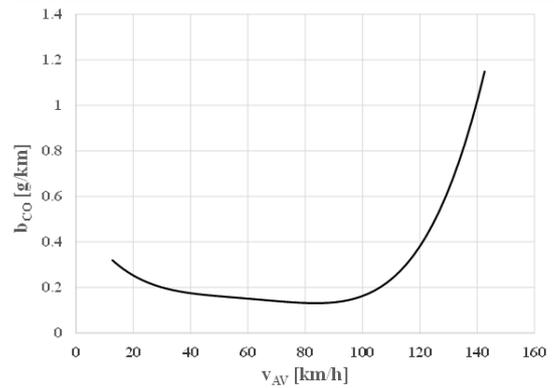


Fig. 9. Dependence of road carbon monoxide emissions on average velocity for the cumulative category of passenger cars in 2020

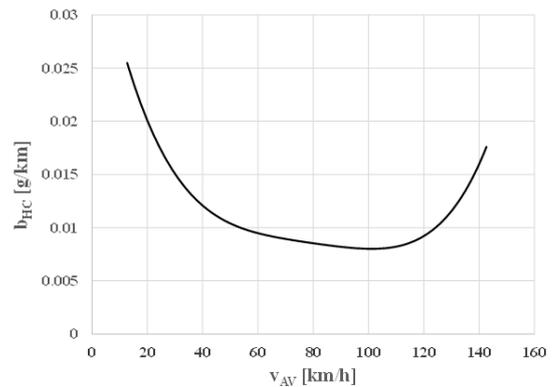


Fig. 10. Dependence of road hydrocarbon emissions on average velocity for the cumulative category of passenger cars in 2020 (own research)

The specific characteristics determined are typical for passenger car engines with respect to pollutant emissions

and fuel consumption [1, 4]. The zero-dimensional characteristics of mass road fuel consumption and road pollutant emissions are presented in Tables 2–5.

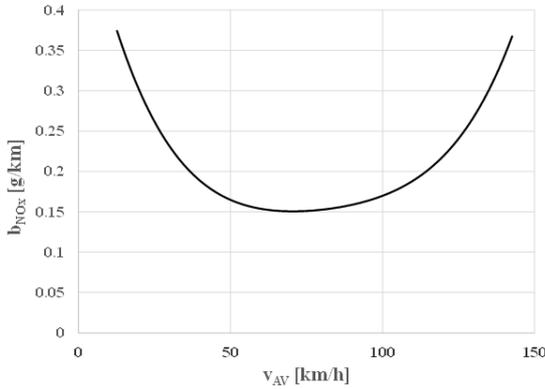


Fig. 11. Dependence of road emissions of nitrogen oxides on the average velocity for the cumulative category of passenger cars in 2020

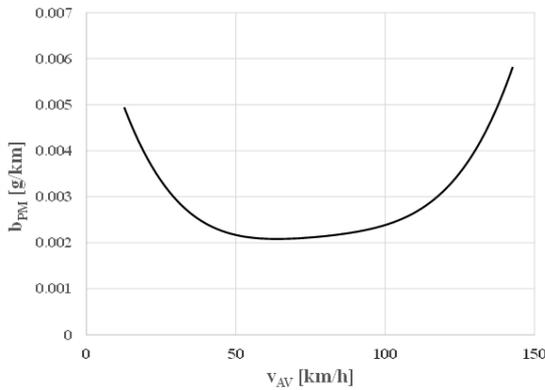


Fig. 12. Dependence of road particulate matter emissions on average velocity for the cumulative category of passenger cars in 2020

Table 2. Zero-dimensional characteristics of mass road fuel consumption and road pollutant emissions in cities in traffic congestions – C

Parameter	1	2	3	4	Min
	[g/km]				
q	95.7	95.9	95.8	96.1	95.7
b _{CO}	0.373	0.374	0.373	0.375	0.373
b _{HC}	0.029	0.029	0.029	0.030	0.029
b _{NOx}	0.426	0.427	0.427	0.428	0.426
b _{PM}	0.006	0.006	0.006	0.006	0.006
b _{CO2}	267.3	267.9	267.7	268.5	267.3
Parameter	Max	Rg	AV	D	W
	[g/km]				
q	96.1	0.4	95.9	0.176	0.002
b _{CO}	0.375	0.002	0.37	0.001	0.002
b _{HC}	0.030	0.000	0.03	0.000	0.002
b _{NOx}	0.428	0.002	0.43	0.001	0.002
b _{PM}	0.006	0.000	0.01	0.000	0.002
b _{CO2}	268.5	1.18	267.8	0.490	0.002

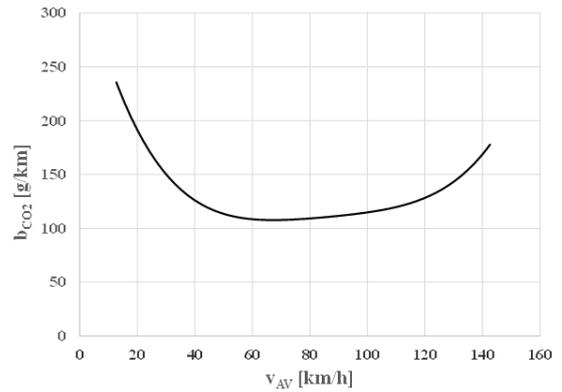


Fig. 13. Dependence of road carbon dioxide emissions on average velocity for the cumulative category of passenger cars in 2020 (own research)

Table 3. Zero-dimensional characteristics of mass road fuel consumption and road pollutant emissions in urban driving conditions outside traffic congestions – U

Parameter	1	2	3	4	Min
	[g/km]				
q	49.9	125.5	125.5	125.5	49.9
b _{CO}	0.183	0.183	0.183	0.183	0.183
b _{HC}	0.013	0.013	0.013	0.013	0.013
b _{NOx}	0.203	0.203	0.203	0.203	0.203
b _{PM}	0.003	0.003	0.003	0.003	0.003
b _{CO2}	134.0	134.1	134.1	134.0	134.0
Parameter	Max	Rg	AV	D	W
	[g/km]				
q	125.5	75.5	106.6	37.76	0.354
b _{CO}	0.183	0.000	0.18	0.000	0.000
b _{HC}	0.013	0.000	0.01	0.000	0.001
b _{NOx}	0.203	0.000	0.20	0.000	0.000
b _{PM}	0.003	0.000	0.00	0.000	0.000
b _{CO2}	134.1	0.13	134.1	0.06	0.000

Table 4. Zero-dimensional characteristics of mass road fuel consumption and road pollutant emissions in non-urban driving conditions – R

Parameter	1	2	3	4	Min
	[g/km]				
q	59.1	125.5	125.5	125.5	59.1
b _{CO}	0.140	0.140	0.141	0.140	0.140
b _{HC}	0.009	0.009	0.009	0.009	0.009
b _{NOx}	0.151	0.151	0.151	0.151	0.151
b _{PM}	0.002	0.002	0.002	0.002	0.002
b _{CO2}	107.8	107.8	107.7	107.8	107.7
Parameter	Max	Rg	AV	D	W
	[g/km]				
q	125.5	66.4	108.9	33.198	0.305
b _{CO}	0.141	0.001	0.140	0.001	0.004
b _{HC}	0.009	0.000	0.01	0.000	0.003
b _{NOx}	0.151	0.000	0.15	0.000	0.000
b _{PM}	0.002	0.000	0.00	0.000	0.001
b _{CO2}	107.8	0.08	107.7	0.04	0.000

The uniformity of average road fuel consumption values is strongest for urban driving and weakest for driving on highways and expressways. Figures 15–19 present the average road pollutant emissions under model traffic condi-

tions for the average velocity of individual implementations and the entire test.

Table 5. Zero-dimensional characteristics of mass road fuel consumption and road pollutant emissions in driving conditions on motorways and expressways – H

Parameter	1	2	3	4	Min
	[g/km]				
q	199.6	125.5	125.5	125.5	125.5
b _{CO}	0.432	0.443	0.408	0.344	0.344
b _{Hc}	0.010	0.010	0.009	0.009	0.009
b _{NOx}	0.229	0.231	0.224	0.211	0.211
b _{PM}	0.003	0.003	0.003	0.003	0.003
b _{CO2}	131.3	131.9	129.9	126.1	126.1

Parameter	Max	Rg	AV	D	W
	[g/km]				
q	199.6	74.1	144.1	37.036	0.257
b _{CO}	0.443	0.099	0.41	0.044	0.109
b _{Hc}	0.010	0.001	0.01	0.000	0.039
b _{NOx}	0.231	0.020	0.22	0.009	0.040
b _{PM}	0.003	0.000	0.00	0.000	0.046
b _{CO2}	131.9	5.838	129.8	2.612	0.020

Figure 14 shows the average value of mass road fuel consumption in model traffic conditions for the average velocity of individual realizations and the entire test.

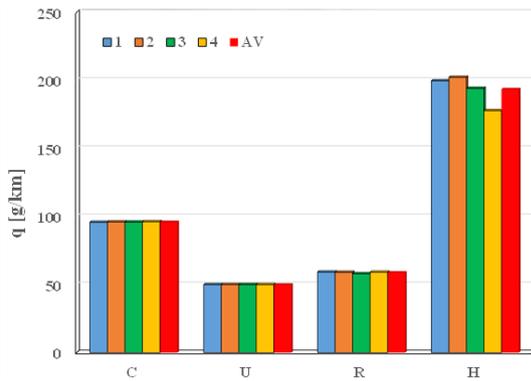


Fig. 14. Average value of mass road fuel consumption – q in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

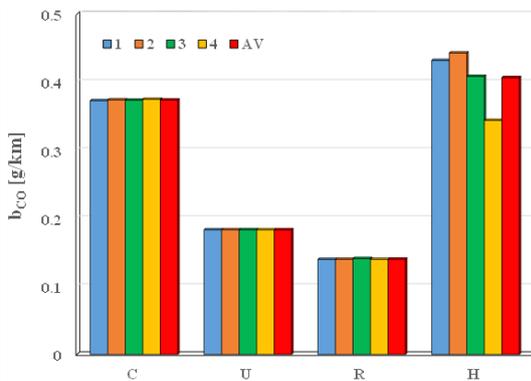


Fig. 15. Average value of road carbon monoxide emissions – b_{CO} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

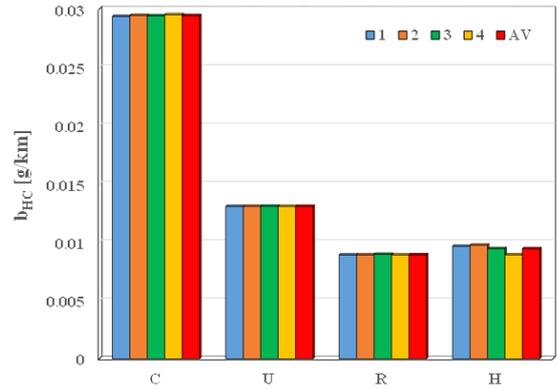


Fig. 16. Average road emissions of hydrocarbons – b_{Hc} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

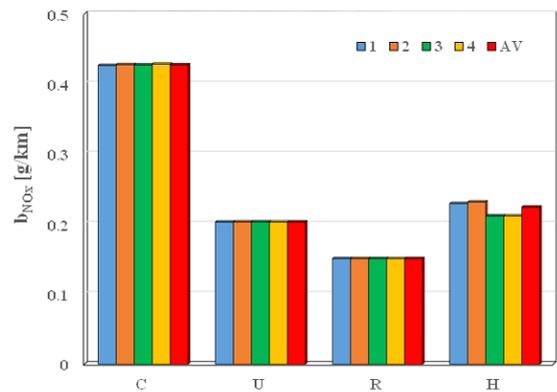


Fig. 17. Average road emissions of nitrogen oxides – b_{NOx} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

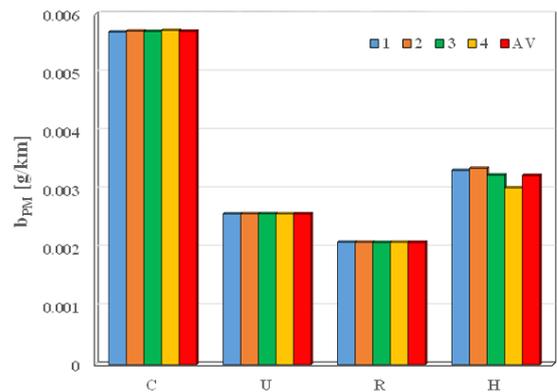


Fig. 18. Average value of road emissions of particulate matter – b_{PM} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

Similarly, for pollutant emissions, uniformity of road emissions is strongest for vehicle traffic in cities and outside cities, and weakest for traffic on highways and expressways. This is confirmed by the coefficients of variation of the average mass road fuel consumption and road emissions of pollutants in individual process implementations under model traffic conditions – Fig. 20–25.

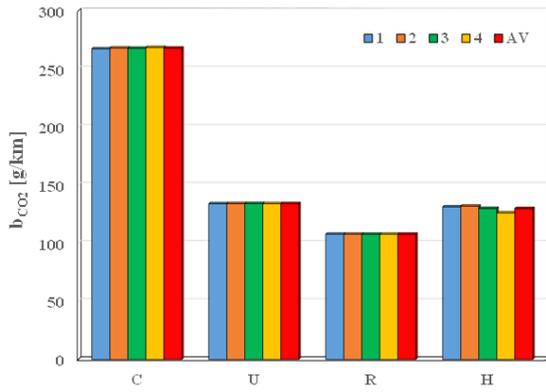


Fig. 19. Average value of road carbon dioxide emissions – b_{CO_2} in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

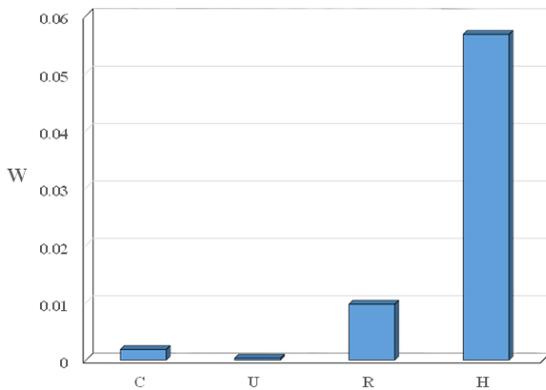


Fig. 20. Coefficient of variation of the average value of mass road fuel consumption in individual process realizations in model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

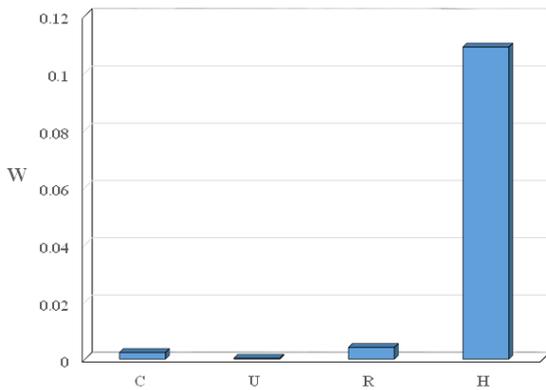


Fig. 21. Coefficient of variation of the average road carbon monoxide emission value in individual process realizations under model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

Figures 26–29 present the coefficients of variation of the average mass road fuel consumption and road pollutant emissions in individual process realizations under model traffic conditions.

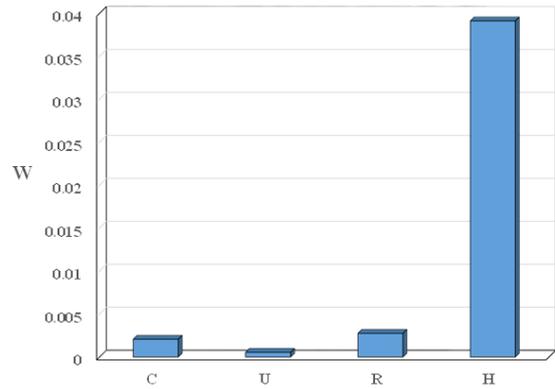


Fig. 22. Coefficient of variation of the average road hydrocarbon emission value in individual process realizations under model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

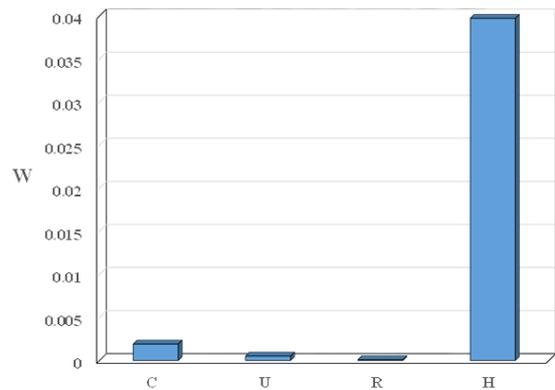


Fig. 23. Coefficient of variation of the average road emission value of nitrogen oxides in individual process realizations under model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

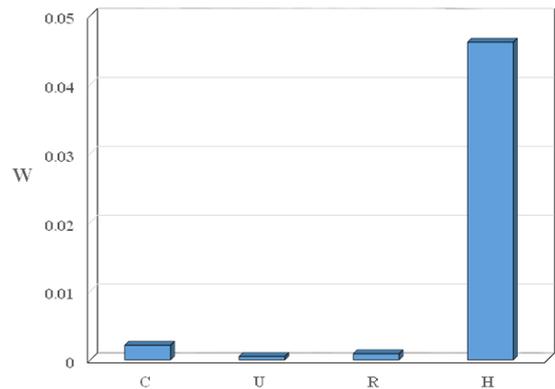


Fig. 24. Coefficient of variation of the average road particulate matter emission value in individual process realizations under model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

The coefficient of variation in modeled traffic conditions and for individual pollutants and fuel consumption varies without any apparent regularity. However, the variation in average road fuel consumption and road pollutant emissions is not significant, demonstrating that fuel con-

sumption and pollutant emissions are not highly sensitive to engine operating states resulting from the modeled stochastic vehicle velocity processes.

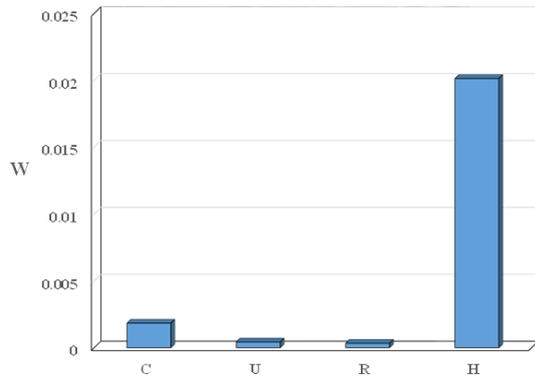


Fig. 25. Coefficient of variation of the average road carbon dioxide emission value in individual process realizations under model traffic conditions in stochastic tests: C – cities in traffic congestions, U – cities outside traffic congestions, R – outside cities, H – on motorways and expressways

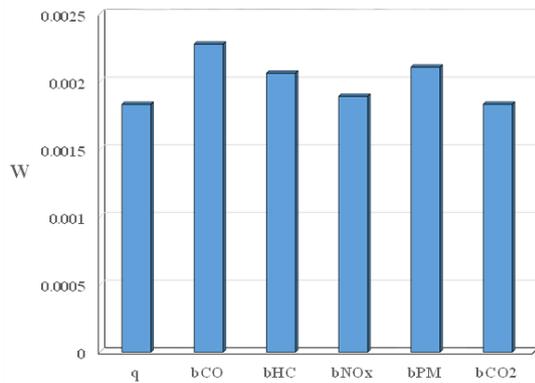


Fig. 26. Coefficient of variation of the average value of mass road fuel consumption – q and road pollutant emissions: carbon monoxide – b_{CO} , hydrocarbons – b_{HC} , nitrogen oxides – b_{NOx} , particulate matter – b_{PM} and carbon dioxide – b_{CO2} in individual process realizations in model traffic conditions in cities in traffic congestions

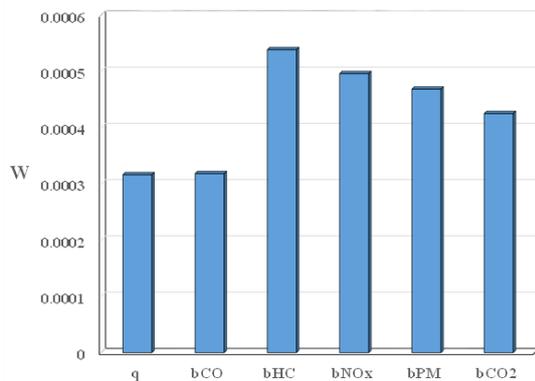


Fig. 27. Coefficient of variation of the average value of mass road fuel consumption – q and road pollutant emissions: carbon monoxide – b_{CO} , hydrocarbons – b_{HC} , nitrogen oxides – b_{NOx} , particulate matter – b_{PM} and carbon dioxide – b_{CO2} in individual process realizations in model traffic conditions in cities outside traffic congestions

The research results and considerations support the thesis that there is a real possibility of using stochastic pro-

cesses as road vehicle velocity models for the purpose of road transport emissions inventory. It is clear that further work is warranted. Above all, it is necessary to establish stochastic tests for other categories of cumulative road vehicles: light duty vehicles, heavy duty vehicles, urban buses, long-distance buses (coaches), and category L vehicles (motorcycles, mopeds, quad bikes, and microcars).

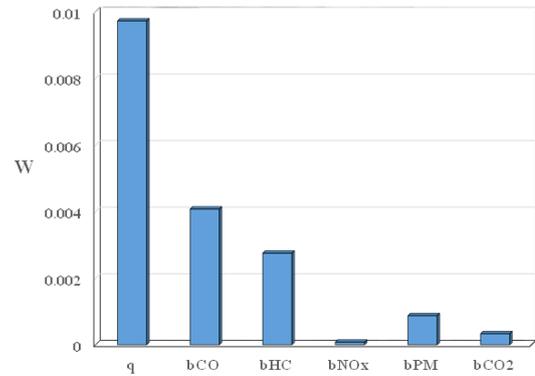


Fig. 28. Coefficient of variation of the average value of mass road fuel consumption – q and road pollutant emissions: carbon monoxide – b_{CO} , hydrocarbons – b_{HC} , nitrogen oxides – b_{NOx} , particulate matter – b_{PM} and carbon dioxide – b_{CO2} in individual process realizations in model traffic conditions outside cities

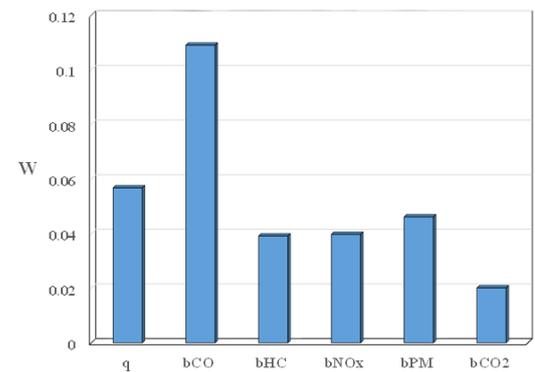


Fig. 29. Coefficient of variation of the average value of mass road fuel consumption – q and road emissions of pollutants: carbon monoxide – b_{CO} , hydrocarbons – b_{HC} , nitrogen oxides – b_{NOx} , particulate matter – b_{PM} and carbon dioxide – b_{CO2} in individual process realizations under model traffic conditions on motorways and expressways

Another research direction is to conduct tests of pollutant emissions and fuel consumption under both chassis dynamometer and real driving conditions using PEMS (Portable Emissions Measurement System) mobile equipment. RDE (Real Driving Conditions) testing could be used to create individual implementations of stochastic road vehicle velocity processes.

5. Summary

Based on the considerations presented in the article, the following conclusions can be drawn:

1. Driving tests based on accurate simulation in the time domain, treated as realizations of stochastic velocity processes of passenger cars under model driving conditions, can be used to effectively study vehicle characteristics in terms of fuel consumption and pollutant emissions. The model's driving conditions include driving in cities with

- traffic congestion, in cities without traffic congestion, outside cities, and on highways and expressways. Vehicle testing under stochastic conditions allows the collection of information not only about vehicle characteristics but also about the statistical features of these characteristics.
- It was found that tests designated as realizations of stochastic velocity processes are characterized by good repeatability of zero-dimensional characteristics for each of the model processes.
 - The results of simulation studies were used as an example to assess the sensitivity of pollutant emissions to pollutant emissions. The characteristics of pollutant emissions from passenger cars for 2020 were determined using simulation methods. The road emissions of pollutants from passenger cars were determined and analyzed under specific realizations of velocity processes under model conditions. The test results were assessed, among other things, for their variability. Good repeatability was found in sets of comparable vehicle traffic conditions.
 - Based on the results, the feasibility of using stochastic processes as road vehicle velocity models for the inventory of road transport pollutant emissions was assessed.
 - Based on the conducted research, it was determined that it would be advisable to continue the work, primarily for other categories of cumulative road vehicles.
 - Another research direction is to conduct pollutant emission and fuel consumption tests under both chassis dynamometer and real driving conditions using mobile PEMS equipment.

Nomenclature

AV	mean value	q	mass fuel consumption
b	specific distance emission	R	outside cities
C	cities in traffic congestions	Rg	range
CT	urban test in traffic congestions	RT	rural test
D	standard deviation	U	cities outside traffic congestions
H	on motorways and expressways	UT	urban test outside traffic congestions
HC	hydrocarbons	W	coefficient of variation
HT	highway test		

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