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Analysis of uncertainty in vehicle collision calculation methods

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This article presents an uncertainty analysis of several of the most commonly used methods for calculating motor vehicle collision speeds. Based on a real-world crash test, a simulation-based collision model and an energy-based post-impact vehicle motion were developed in MATLAB. The calculation results were optimized using the Monte Carlo method. A custom-designed script was employed to search the full range of feasible solutions, and the results were compared with those obtained from analytical reconstruction methods and simulations conducted in two commercially available vehicle dynamics software packages. The study demonstrated that analytical calculations are highly sensitive to uncertainties in estimating the base post-impact trajectory angles and, depending on the reconstruction type (I, II, III, IV), to pre-impact approach angles. This sensitivity is not observed in simulation-based calculations, regardless of whether the post-impact motion is modeled using energy methods or vehicle dynamics models integrated into accident reconstruction software. Additionally, the performance of optimization algorithms built into these programs was analyzed. In this test, the uncertainties for the different methods – at measured speeds of 42.1 and 30.0 km/h – did not exceed ±13%. In contrast, randomizing the friction coefficient (μ) resulted in uncertainties of up to ±28% and varied across individual vehicles. These values are consistent with the theoretical analyses conducted by other researchers.

Key words: *vehicle collision, reconstruction calculations, simulation-based calculations, computational uncertainty, energy equivalent speed (EES)*

1. Introduction and aim of the study

The process of recreating road traffic incidents is commonly referred to as reconstruction [13]; however, from an engineering perspective, a distinction is made between reconstruction calculations and simulation-based calculations. Theoretical reconstruction calculations can be categorized into four fundamental types, where the selection of the appropriate model depends on the available data set, such as information regarding post-impact rotational motion, velocity vector angles, etc. The specific nature of theoretical reconstruction calculations – reflected in the mathematical forms of physical dependencies (e.g., trigonometric functions) – can lead to potentially significant computational uncertainties, sometimes exceeding 100%. These methods are highly sensitive to the angles describing the positions of the vehicles' center-of-mass velocity vectors, both at the moment of impact and immediately thereafter. Post-impact motion poses a particular challenge due to limited data availability, such as the absence of tire marks [14]. Another reconstruction method, though characterized by significantly lower uncertainty, involves calculating collisions with fixed objects (e.g., poles or trees), using empirical mathematical models [16]. Reconstruction calculations do not allow visualization of post-impact trajectories, and a single calculation run can give a misleading sense of accuracy. Therefore, it is advisable to perform multiple iterations with varying approaches and departure angles to assess the uncertainty specific to a given case.

Such uncertainties are not characteristic of computational methods implemented in simulation environments [6, 8]. These environments utilize iterative vehicle dynamics models to visualize post-impact trajectories, as demonstrated in this study using two dedicated accident reconstruction software packages: V-Sim [1, 15] and PC-Crash [10, 11]. However, simulation-based calculations can also be performed without a full vehicle dynamics model by modeling post-impact motion through energy-based relationships (e.g., Marquard, Burg, or McHenry-Marquard [16]). These models incorporate correction factors for non-parallel wheel reaction force vectors (empirical models) that account for rotational motion and sideslip angles [7, 8], although they do not model post-impact trajectories sensu stricto. This approach was implemented in the proprietary MATLAB algorithm presented in this paper.

The most advanced methods for vehicle collision calculations are based on the Finite Element Method (FEM) [2, 4, 5, 18]. However, these are primarily used to determine the EES (Equivalent Energy Speed) parameter, which does not represent the actual impact speed, but rather the energy of deformation work expressed in speed units. Consequently, this parameter is subsequently used as an input in some of the calculation types presented in this study. FEM methods are not commonly used by forensic experts to reconstruct impact speeds in road accidents because they are time-consuming and resource-intensive. A significant drawback is also the lack of FEM models for the wide range of vehicles used in road traffic. Simulation software used by experts typically contains only a few vehicle FEM models, mostly of American origin. These methods are primarily used for research purposes and, on rare occasions, to solve collision problems involving atypical obstacles, which are individually modeled based on specific strength test results.

It must be emphasized that the "reliability" of accident reconstruction calculations is inversely proportional to the computational uncertainty, given the unique, non-repeatable nature of a single collision. Consequently, the probability of the occurrence of a single, precise speed value tends toward zero [17]. This perspective is dictated by legal requirements in judicial proceedings, where the range of uncertainty often determines the range of accident-avoidance possibilities and, thus, the assignment of liability [12].

In light of the above, the aim of this study was to investigate the uncertainty of calculated impact speeds using the aforementioned methods, based on a real-world crash test.

The authors were unable to identify any studies that compare the results of several different methods based on a single real-world crash test. Existing literature focuses on the uncertainty of a specific method used by researchers for a particular test; therefore, a reliable comparison of uncertainties across methods is not feasible, as different crash tests were used in each case. While study [2] presented a comparison of method uncertainties, it was based on theoretical analysis rather than specific crash tests. Their findings for various methods were as follows: limit values ±30.5%, total derivative ±31.3%, second-order total derivative ±33.0%, finite differences ±31.3%, Gaussian probabilistic method ±19.0%, stochastic process probabilistic method ±18.8%, and Monte-Carlo probabilistic method ±29.5%.

Consequently, this constitutes the innovative contribution of the present publication. Nevertheless, establishing a comprehensive view of the uncertainties inherent in individual methods would require numerous analogous analyses, which remains a goal for future research.

2. Research object – crash test

2.1. Vehicles

The crash test data was retrieved from the PC-Crash software database. The collision involved two vehicles: a Ford Mondeo and an Opel Vectra, as shown in Fig. 1. The fundamental technical parameters of both vehicles are presented in Table 1.

Fig. 1. View of the vehicles selected for the study: Ford (1) with an initial speed of 42.1 km/h and Opel (2) with an initial speed of 38.0 km/h

Table 1. Vehicle parameters

No.	Make, model, type	Ford (1) Mondeo MK I	Opel (2) Vectra Bi
1.	impact speed [km/h]	42.1	38.0
2.	year of manufacture	1995	1991
3.	curb weight [kg]	1306	1170
4.	length [m]	4.48	4.48
5.	width [m]	1.75	1.70
6.	height [m]	1.50	1.42
7.	wheelbase [m]	2.70	2.60
8.	track width [m]	1.50	1.50
9.	distance CoG to front axle [m]	1.03	1.09
10.	CoG height [m]	0.56	0.50

2.2. Modeling of the collision and post-impact motion

The relative positioning of the vehicles at the moment of impact, the distances traveled during post-impact motion, and their final rest positions are presented in Fig. 2.

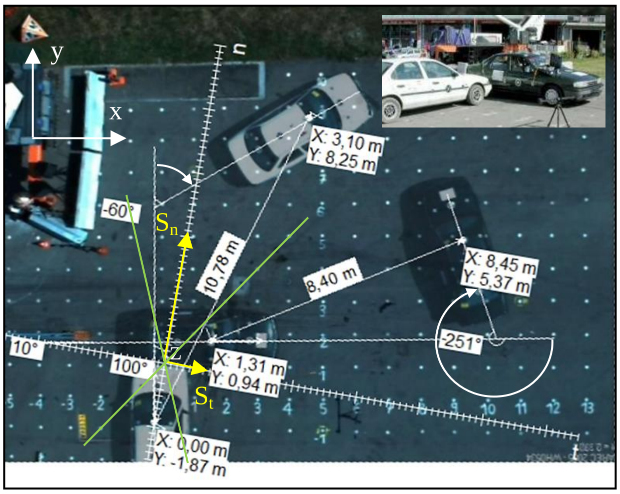


Fig. 2. Impact and final rest positions in the normal (n) and tangential (t) coordinate system relative to the vehicle deformation surface, with the global coordinate system (x, y) overlaid. Shown are the impulses components S_t and S_n (yellow) and the friction cone (green)

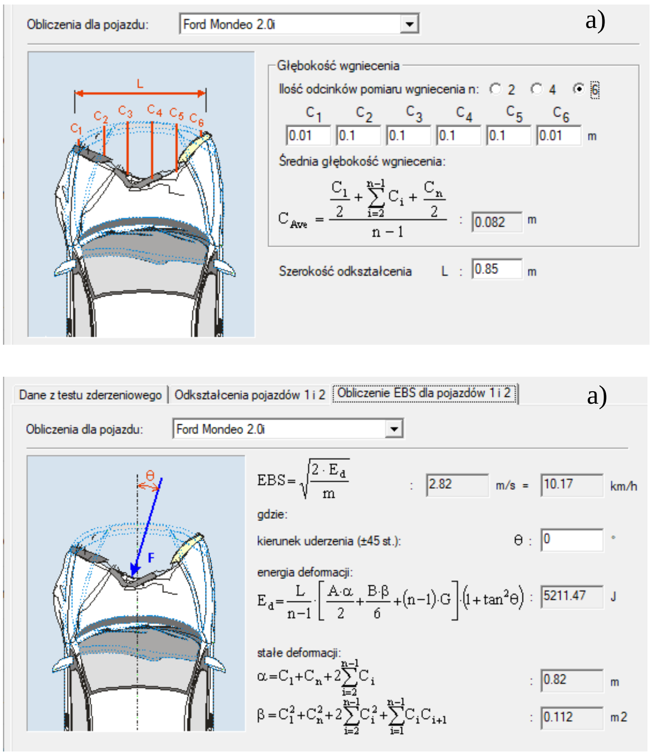
In simulation-based calculations, distances and angles are defined in the normal and tangential impact plane coordinate system, with the origin at the point of application of the collision impulse. The ratio of the tangential impulse component S_t to the normal component S_n is defined as the friction at the collision node and is described by the predefined parameter tgρ (friction cone). If the S_t/S_n ratio equals or exceeds the given value of tgρ (where the collision impulse lies on the surface of the friction cone), a so-called rough collision with sliding occurs; otherwise, it is classified as a rough collision without sliding. Furthermore, simulation-based calculations incorporate a predefined coefficient of restitution, k, that determines the amount of energy dissipated during the impact. This coefficient is non-linearly and inversely proportional to the relative velocity of the vehicles along the impact normal. Consequently, when modeling the collision process, it is possible to influence the relative pre-impact angles, the location of the impulse application point within the contact area, the friction at the collision node, the orientation of the tangential plane, and the vehicle rebound reflected in the coefficient of restitution [6, 8]. All these parameters affect the post-impact trajectory angles and the overall dynamics of the vehicles.

Conversely, in reconstruction calculations, the reference system is the global coordinate system rather than the normal and tangential planes. These methods do not utilize the parameter or the coefficient *tgρ* of restitution *k*. Therefore, one can only influence the pre- and post-impact motion angles. However, post-impact angles are difficult to determine with sufficient precision, and even minor variations in them can significantly affect the calculation results [14].

2.3. Energy equivalent speed (EES)

In both types of calculations (simulation-based and reconstruction), the Energy Equivalent Speed (EES) parameter is utilized. It should not be equated with the actual vehicle speed, except in one specific scenario: when a vehicle strikes a rigid, non-deformable barrier and no significant post-impact motion occurs. The EES parameter is closely related to the Equivalent Barrier Speed (EBS); however, EBS also accounts for the vehicle body's elastic work. This elastic component can typically be neglected, as it usually does not exceed at very low speeds (e.g. 10 km/h) and becomes negligible at higher speeds, which results from the velocity-dependent characteristics of the coefficient of restitution [16].

In simulation-based calculations, the EES parameter is used as a verification (control) value, whereas in reconstruction calculations it serves as an input value, though not in all such calculations. For the purpose of this article, the EES value (which, in this case, is very similar to the EBS) was calculated according to the CRASH3 standard using the RWD software [20], as schematically shown in Fig. 3.



Obliczenia dla pojazdu: Ford Mondeo 2.0i

Głębokość wgniecenia

Ilość odcinków pomiaru wgniecenia n: ○ 2 ○ 4 ● 6

C_1	C_2	C_3	C_4	C_5	C_6	
0.01	0.1	0.1	0.1	0.1	0.01	m

Średnia głębokość wgniecenia:

$$C_{Ave} = \frac{\frac{C_1}{2} + \sum_{i=2}^{n-1} C_i + \frac{C_n}{2}}{n-1}$$: 0.082 m

Szerokość odkształcenia L : 0.85 m

Dane z testu zderzeniowego | Odkształcenia pojazdów 1 i 2 | Obliczenie EBS dla pojazdów 1 i 2

Obliczenia dla pojazdu: Ford Mondeo 2.0i

$$EBS = \sqrt{\frac{2 \cdot E_d}{m}}$$: 2.82 m/s = 10.17 km/h

gdzie:

kierunek uderzenia (±45 st.): θ : 0 °

energia deformacji:

$$E_d = \frac{L}{n-1} \cdot \left[\frac{A \cdot \alpha}{2} + \frac{B \cdot \beta}{6} + (n-1) \cdot G \right] \cdot \left(1 + \tan^2 \theta\right)$$: 5211.47 J

stałe deformacji:

$$\alpha = C_1 + C_n + 2\sum_{i=2}^{n-1} C_i$$: 0.82 m

$$\beta = C_1^2 + C_n^2 + 2\sum_{i=2}^{n-1} C_i^2 + \sum_{i=1}^{n-1} C_i C_{i+1}$$: 0.112 m2

Fig. 3. Input data and calculation results – example for vehicle (1)

For the second vehicle, the EBS was approximately 13.6 km/h.

3. Methodology – simulation-based calculations

3.1. Limitations of the current objective function

The research workstation consisted of a PC equipped with specialized software: V-Sim, PC-Crash, and RWD. To ensure the comparability of results, the Kudlich-Slibar collision model [14] was employed in both primary programs. Although these software packages include alternative collision models (two in PC-Crash and one in V-Sim), those models offer limited adjustability, and the user's ability to influence their parameters is practically restricted.

The calculations were conducted using two distinct approaches. First, the built-in optimizer of each program was utilized. Subsequently, the data was cleared from memory, and the collision was calculated in manual mode. This procedure was necessary because built-in optimizers often function incorrectly due to flawed objective functions (quality functions). As demonstrated in [9], the objective function in PC-Crash [10], based on the principle of weighted aggregate criteria, combines parameters that are incomparable (e.g., angles, distances, EES). Consequently, the percentage error (uncertainty) calculated by the software does not correlate with the collision energy balance and thus does not represent the actual percentage uncertainty of the speed calculation. In the case of V-Sim, the authors have not disclosed the specific form of the fitness function, although the input parameters and resulting outputs suggest it is more advanced than the one in PC-Crash. Instead of a percentage error, the user receives a range of calculated speeds that satisfy predefined post-simulation motion criteria.

Nevertheless, situations still occur where the calculation results are physically unrealistic, or it is impossible to achieve post-impact vehicle positions consistent with the test documentation. In PC-Crash, it is sometimes impossible to optimize (find a solution for) a virtual collision – even one previously designed in manual mode within the same program – once the sought-after data has been reset [9]. While the vehicle models in these programs are known approximations of reality, it is understandable that they may struggle to optimize a real-world crash test. However, the algorithm should not encounter such difficulties when solving a virtual test generated within its own environment.

The criterion for the correctness of the simulation-based calculations is the achievement of post-impact (post-simulation) positions by the virtual vehicles. These positions should align as closely as possible with the actual rest positions, as the vehicles' kinetic energy is depleted during both planar and spatial motion.

3.2. Simulation-based calculations in V-Sim

The results obtained using the optimization module are presented in Fig. 4.

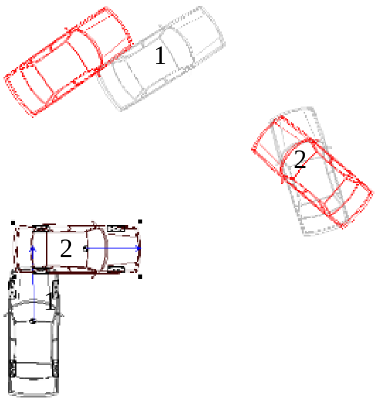


Fig. 4. V-Sim optimizer calculations (matching post-simulation positions to real-world data)

In 'manual' mode (without the use of the software's built-in optimizer), the simulation results were successfully improved, as demonstrated in Fig. 5.

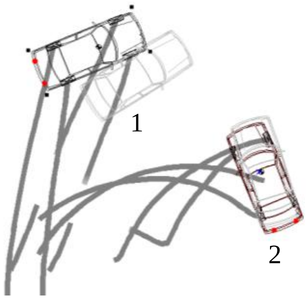


Fig. 5. V-Sim manual calculations (matching post-simulation positions to real-world data)

In both calculation cases, by appropriately selecting the input forces and accounting for the tire-road friction coefficient (μ), the delayed braking of the Ford's wheels (observed in the test video) and the lift-off of the Opel's left wheels were modeled. The calculated speed results will be presented collectively at the end of the paper.

3.3. Simulation-based calculations in PC-Crash

The use of the built-in optimizer yielded the results shown in Fig. 6.

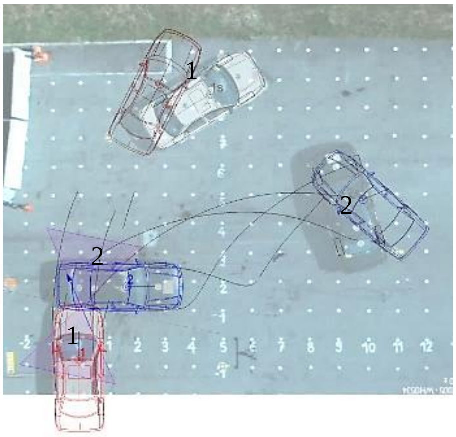


Fig. 6. PC-Crash optimizer calculations (matching post-simulation positions to real-world data)

During the calculations, the software's optimization algorithm tended to converge on local minima, resulting in unrealistic impact speed values. Furthermore, the simulated vehicles came to a rest far from their target positions. In 'manual' mode, the simulation results were significantly improved, achieving a better fit to the final rest positions than in V-Sim, as shown in Fig. 7.

In contrast to V-Sim, manual calculations using the Kudlich-Slibar model are significantly more efficient in PC-Crash, as adjustments to the impulse application point and other parameters can be made without closing the navigation window while maintaining overall stability. This may be because the software was primarily designed for the force-based collision method [15]. However, this force-based model is fully automated and does not allow for user intervention. Consequently, it is impossible to model the observed immediate post-impact motion resulting from, for instance, shifting the impulse application point (due to variable body stiffness at different structural locations) or varying the friction at the collision node by rotating the tangential plane. Therefore, the Kudlich-Slibar model remains the most practical choice for such analyses.

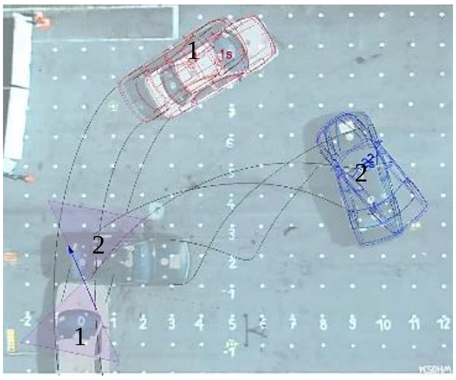


Fig. 7. PC-Crash manual calculations (matching post-simulation positions to real-world data)

4. Methodology – type I analytical reconstruction calculations

Due to the orthogonal directions of the vehicles' pre-impact velocity vectors, Type I analytical reconstruction calculations based on the principle of conservation of momentum were applied. This approach is schematically presented in Fig. 8.

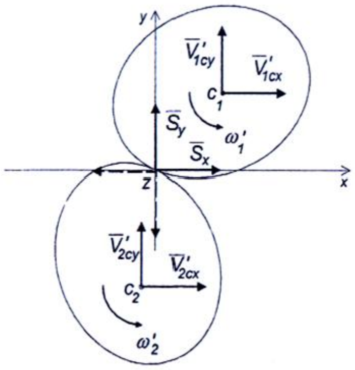


Fig. 8. Parameter designations for reconstruction calculations [14]

After appropriate transformation, the equations take the following form:

$$V_1 = \frac{m_1 \cdot V'_1 \cdot \sin(\alpha_2 - \alpha'_1) + m_2 \cdot V'_2 \cdot \sin(\alpha_2 - \alpha'_2)}{m_1 \cdot \sin(\alpha_2 - \alpha_1)} \tag{1}$$

$$V_2 = \frac{m_1 \cdot V'_1 \cdot \sin(\alpha'_1 - \alpha_1) + m_2 \cdot V'_2 \cdot \sin(\alpha'_2 - \alpha_1)}{m_2 \cdot \sin(\alpha_2 - \alpha_1)} \tag{2}$$

where: α – the pre-impact velocity vector angle (immediately before impact), α' – the post-impact velocity vector angle (immediately after impact).

Post-impact speeds were calculated using Marquard's energy-based relationships [16] for braking distance, incorporating empirical correction factors:

$$k_s = 0.17 \cdot k^3 - 0.488 \cdot k^2 - 0.03 \cdot k + 1 \tag{3}$$

$$k_\varphi = 0.328 \cdot k^3 - 0.772 \cdot k^2 + 1.072 \cdot k \tag{4}$$

$$k = \left| \frac{\Delta\varphi \cdot l}{2 \cdot s} \right| \tag{5}$$

where k in this relationship denotes the rotation factor rather than the coefficient of restitution, l – vehicle length.

Since the post-impact motion angles were well documented in the crash test, the calculations had low uncertainty. However, it was observed that even minor variations in the departure angles had a drastic impact on the results. In contrast, real-world traffic incidents are typically not as well documented as they are in video footage.

5. Methodology – optimization-based simulation calculations in MATLAB

The Kudlich-Slibar collision model [14] was implemented within the proprietary optimization script, with its fundamental relationships defined as follows:

$$m_1 \cdot (V'_{1ct} - V_{1ct}) = S_t \tag{6}$$

$$m_1 \cdot (V'_{1cn} - V_{1cn}) = S_n \tag{7}$$

$$m_2 \cdot (V'_{2ct} - V_{2ct}) = -S_t \tag{8}$$

$$m_2 \cdot (V'_{2ct} - V_{2ct}) = -S_t \tag{9}$$

$$I_{1c} \cdot (\omega'_1 - \omega_1) = S_t \cdot n_{1c} - S_n \cdot t_{1c} \tag{10}$$

$$I_{2c} \cdot (\omega'_2 - \omega_2) = S_n \cdot t_{2c} - S_t \cdot n_{2c} \tag{11}$$

The quantities n_{1c},t_{1c},n_{2c},t_{2c} (impulse moment arms – see Fig. 2 and Fig. 9) represent the geometric coordinates of the nominal contact point Z relative to the center of mass of each body.

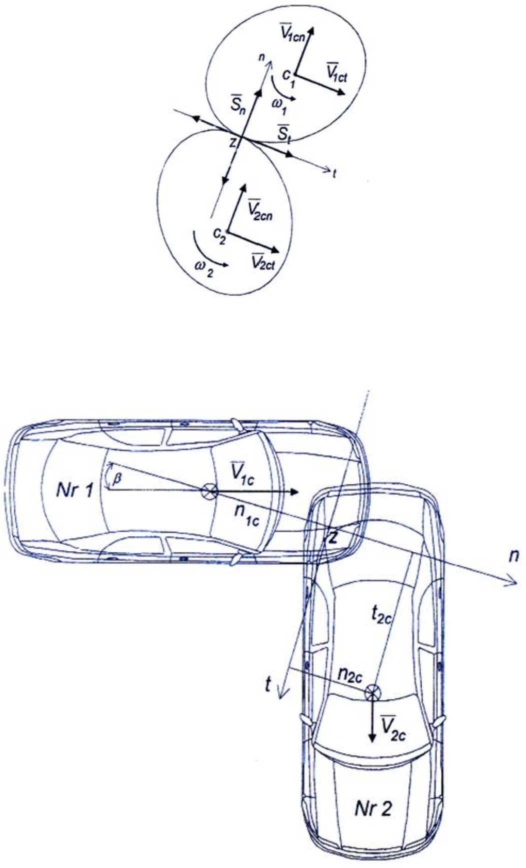


Fig. 9. Parameter designations for simulation-based calculations [14]

The post-impact motion was modeled according to Marquard's method, with the braking distance relationships transformed from a reconstruction (backward) to a simulation (forward) form. Consequently, the input data consisted of the post-impact speeds derived from the collision simulations, along with the known distances and angles of the post-impact motion. The calculation criterion (convergence condition) was satisfied when the speeds at the end of the post-impact phase reached values near zero, indicating that the vehicles' kinetic energy had been sufficiently dissipated. The objective (optimized) values were the pre-impact speeds, which were sampled using a uniform distribution via the Monte Carlo method. The proprietary script continued to iterate until a predefined number of speed pairs satisfying the velocity reduction condition in the post-impact motion were collected. This process can be described by the following objective function, which serves as the equivalent of Marquard’s relationship:

$$f(V_k) = \sqrt{V'^2 - 2 \cdot ks \cdot \mu \cdot g \cdot s} \tag{12}$$

$$0 < V_k < 0.5 \left[\frac{m}{s}\right] \tag{13}$$

The proprietary algorithm also incorporates the capability to randomize (search for) the friction coefficient (μ), reflecting the utilization of braking forces. This approach yielded a three-dimensional relationship between the pre-impact speeds of both vehicles and their post-impact decelerations, as shown in Fig. 10.

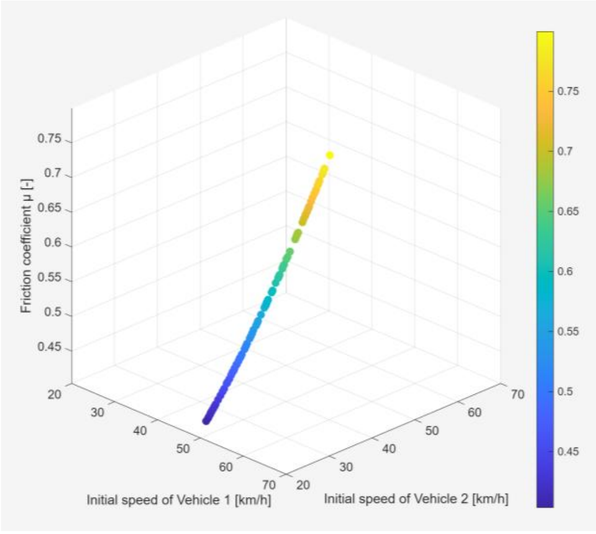


Fig. 10. 3D view of initial vehicle speeds as a function of the friction coefficient (μ) within the range of 0.4 to 0.8

The interdependencies between the calculated impact speeds are presented in Fig. 11.

The relationship, indicating that higher post-impact motion resistance coefficients lead to higher calculated pre-impact speeds, demonstrates the forensic importance of examining the full uncertainty range within physically realistic limits. Post-impact motion can be highly complex; furthermore, it is impossible to precisely reconstruct the actual resistance coefficient at the moment of the incident for the specific vehicles involved. In most cases, further empirical research is no longer possible, as the vehicles have been destroyed and a significant amount of time has elapsed since the event [12].

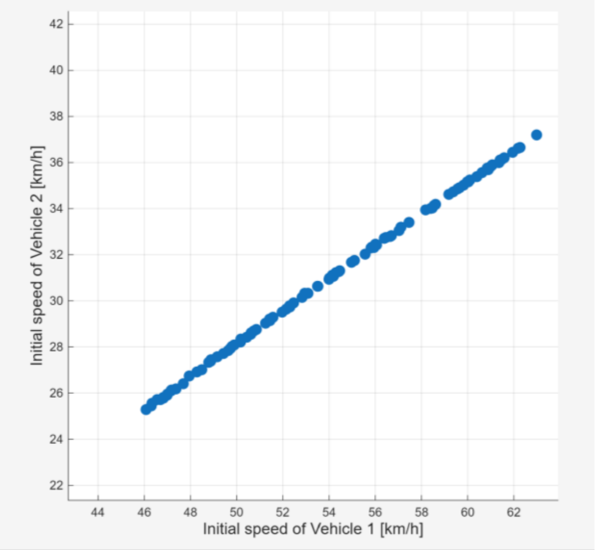


Fig. 11. 2D view of initial vehicle speeds as a function of the friction coefficient (μ) within the range of 0.4 to 0.8

6. Summary of results

During the calculations performed using all methods, it was observed that the speed of vehicle No. 1 (Ford Mondeo) was significantly higher than the value measured during the crash test. However, there is no evidence to suggest that the test measurement was erroneous; therefore, this discrepancy must be attributed to modeling uncertainty and the assumed coefficients – for instance, the tire-road friction utilization, even though the values selected were as low as possible while remaining physically realistic.

The chart in Fig. 12 presents pre-impact speed values calculated using various methods. The horizontal lines represent the mean values; however, calculations from the proprietary MATLAB script were excluded from the overall mean. This is because those results themselves represent the average values derived from 100 solutions that satisfied the criteria. This approach was taken to avoid the "mean of means" bias and to allow for a direct comparison between the averages of individual methods and the mean values generated by the proprietary script. The final two bars of the chart refer to the additional optimization of the friction coefficient (μ), corresponding to the plots shown in Fig. 10 and Fig. 11. The larger deviation of the mean in the case of μ-randomization results from a significant expansion of the speed range satisfying the criteria. Although the μ values were sampled from a range of 0.4 to 0.8, it appears that in this specific case, values closer to the lower bound were more appropriate. Nonetheless, in a real-world traffic incident – lacking a precise video recording – it would be impossible to narrow this range, making all resulting values equally probable.

As mentioned in the introduction, the authors were unable to identify any studies that compare commonly used forensic methods across a single specific crash test or a group of tests. While such comparisons are likely performed during the preparation of expert opinions and forensic reports – as verifying results with a second method is considered best practice (though rarely using all available methods) – these findings are extremely seldom published. An additional complication is the frequent lack of a 'ground truth' reference for actual speed; even when Event Data Recorder (EDR) records are available, they must be verified using independent methods based on other physical evidence. Consequently, in forensic reports, uncertainty is estimated while the absolute true value remains unknown. For instance, if a vehicle is skidding sideways or has locked wheels, the recorded EDR speed may not reflect the actual velocity. The same applies to acceleration with loss of traction.

Since the methods utilized in this study remain standard in forensic accident reconstruction, it is essential to continue research on various crash tests to obtain a broader understanding of uncertainty across different collision types. Furthermore, it is critical to acknowledge that the calculated impact speed is mathematically dependent on post-impact movement parameters, which are typically burdened by high partial uncertainty. Moreover, the pre-impact resistance is often unknown, creating additional difficulties in establishing the initial speed at the onset of danger – the parameter of greatest interest to judicial authorities.

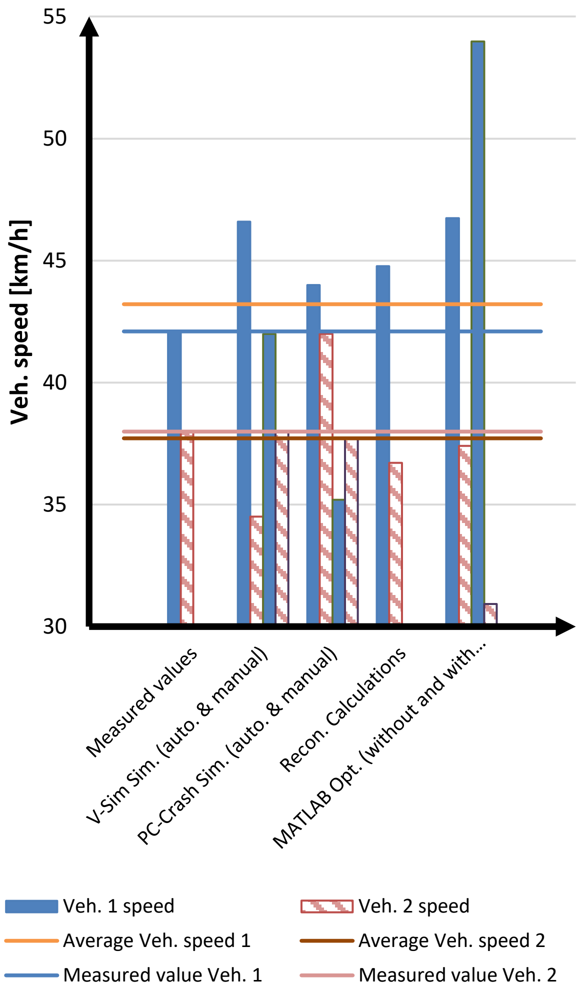


Fig. 12. Impact speeds calculated using various methodologies

In general, for this specific test at measured speeds of 42.1 and 30.0 km/h, the uncertainties for the various methods did not exceed ±13%. In contrast, randomizing the friction coefficient (μ) resulted in a maximum uncertainty of ±28%, with results varying across vehicles. These values fall within the range of theoretical analyses by other researchers [2], who obtained the following results: limit values ±30.5%, total derivative ±31.3%, second-order total derivative ±33.0%, finite differences ±31.3%, Gaussian probabilistic method ±19.0%, stochastic processes ±18.8%, and Monte-Carlo simulation ±29.5%.

7. Conclusions

The computational analysis, conducted using available methodologies and further enriched by calculations performed with a proprietary MATLAB script, has led to the following conclusions.

1. Objective functions in simulation software allow for moderately effective matching of post-impact vehicle positions; however, they provide the capability to randomize input parameters, thus enabling uncertainty analysis in a qualitative sense. The objective function in PC-Crash lacks physical significance in terms of energy balance; consequently, the percentage error reported by the software does not correlate with the actual speed calculation error, although it may be misinterpreted as such. In contrast, V-Sim does not report a percentage error; instead, it provides only the number of results that satisfy the defined constraints. This approach does not mislead the user regarding the actual uncertainty of the results.
2. The collision simulation designed in MATLAB, based on Kudlich-Slibar relationships and Marquard-based post-impact motion, enables rapid assessment of the solution's consistency with observed collision effects and the investigation of uncertainty ranges for various parameters. However, it lacks visualization capabilities and does not account for post-impact trajectories sensu stricto, which in certain cases represents a significant limitation.
3. A comparison of the results obtained through various methodologies—including the analytical reconstruction model, simulation-based calculations in V-Sim and PC-Crash, and the proprietary MATLAB script – demonstrated their mutual consistency. From the perspective of road accident reconstruction, the results showed sufficient alignment with real-world crash test data, thereby validating their practical applicability.
4. Analytical reconstruction calculations demonstrated that even minor deviations in the input data (a few degrees in the approach and departure angles) can drastically affect the resulting speeds (by 100% or more), confirming a well-known shortcoming of this methodology. The computational uncertainty determined in this study remained low only because the precise pre- and post-impact vehicle motion was known – specifically, the trajectory angles immediately following the collision
5. While simulation-based calculations may exhibit sensitivity in certain cases, they are generally significantly less sensitive to uncertainties regarding the aforementioned angles than analytical methods
6. Generally, for this specific test, the uncertainties of the various methods did not exceed a maximum of ±13%, whereas the randomization of the friction coefficient (μ) resulted in a maximum uncertainty of ±28%, with results varying for each vehicle. These uncertainty levels were consistent with values reported by other authors who have investigated the problem from a theoretical perspective. For other collision configurations and conditions, these values may differ.

Nomenclature

E	energy	m	mass
EES	equivalent energy speed	s	distance
EBS	equivalent barrier speed	tgρ	friction coefficient at the collision node
k	coefficient of restitution, or Marquardt's rotation coefficient	μ	coefficient of friction of the wheels on the road

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